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SECTION 9.4

Complex Numbers

OBJECTIVES

1. Express square roots of negative numbers in terms of i.
2. Write complex numbers in $a + bi$ form.
3. Add and subtract complex numbers.
4. Multiply and divide complex numbers.
5. Solve quadratic equations with complex-number solutions.

ARE YOU READY?

The following problems review some basic skills that are needed when working with complex numbers.

1. Explain why $\sqrt{-16}$ is not a real number.
2. Add: $(2x + 5) + (3x + 7)$
3. Subtract: $(x - 1) - (4x + 8)$
4. Multiply: $9x(5x + 6)$
5. Multiply: $(2x + 3)(7x - 1)$
6. Solve: $x^2 - 24 = 0$

The Language of Algebra

For years, mathematicians thought numbers like $\sqrt{-4}$ and $\sqrt{-23}$ were useless. In the 17th century, René Descartes (1596–1650) referred to them as **imaginary numbers**. Today they have important uses in electronics, engineering, and even biology.

Earlier in this chapter, we saw that some quadratic equations do not have real-number solutions. When we used the square root property or the quadratic formula to solve them, we encountered the square root of a negative number.

***Section* 9.1 *Example* 1d**

Solve: $n^2 = -4$

$n = \pm\sqrt{-4}$

No real-number solutions

***Section* 9.3 *Example* 4**

Solve: $x^2 - x + 6 = 0$

$x = \dfrac{1 \pm \sqrt{-23}}{2}$

No real-number solutions

To solve such equations, we must define the square root of a negative number.

1 Express Square Roots of Negative Numbers in Terms of *i*.

Recall that the square root of a negative number is not a real number. However, an expanded number system, called the *complex number system,* has been devised to give meaning to expressions such as $\sqrt{-4}$ and $\sqrt{-23}$. To define complex numbers, we use a new type of number that is denoted by the letter i.

The Number *i*

The imaginary number i is defined as

$$i = \sqrt{-1}$$

From the definition, it follows that $i^2 = -1$.

We can use extensions of the product and quotient rules for radicals to write the square root of a negative number as the product of a real number and i.

EXAMPLE 1 Write each expression in terms of i: **a.** $\sqrt{-4}$ **b.** $\sqrt{-23}$ **c.** $-\sqrt{-48}$ **d.** $\sqrt{-\dfrac{54}{25}}$

Strategy We will write each radicand as the product of -1 and a positive number. Then we will apply the appropriate rules for radicals.

Why We want our work to produce a factor of $\sqrt{-1}$ so that we can replace it with i.

Solution After factoring the radicand, use the product rule for radicals.

a. $\sqrt{-4} = \sqrt{-1 \cdot 4} = \sqrt{-1}\sqrt{4} = i \cdot 2 = 2i$ Replace $\sqrt{-1}$ with i.

b. $\sqrt{-23} = \sqrt{-1 \cdot 23} = \sqrt{-1}\sqrt{23} = i\sqrt{23}$ or $\sqrt{23}i$ Replace $\sqrt{-1}$ with i.

c. $-\sqrt{-48} = -\sqrt{-1 \cdot 16 \cdot 3} = -\sqrt{-1}\sqrt{16}\sqrt{3} = -i \cdot 4 \cdot \sqrt{3} = -4i\sqrt{3}$ or $-4\sqrt{3}i$

d. After factoring the radicand, use the product and quotient rules for radicals.

$$\sqrt{-\frac{54}{25}} = \sqrt{-1 \cdot \frac{54}{25}} = \frac{\sqrt{-1 \cdot 54}}{\sqrt{25}} = \frac{\sqrt{-1}\sqrt{9}\sqrt{6}}{\sqrt{25}} = \frac{3i\sqrt{6}}{5} \text{ or } \frac{3\sqrt{6}}{5}i$$

Notation

Since it is easy to confuse $\sqrt{23i}$ with $\sqrt{23}i$, we write i first so that it is clear that the i is not within the radical symbol. However, both $\sqrt{23}i$ and $i\sqrt{23}$ are correct.

Self Check 1 Write each expression in terms of i: **a.** $\sqrt{-81}$ **b.** $-\sqrt{-11}$ **c.** $\sqrt{-28}$ **d.** $\sqrt{-\frac{27}{100}}$

Now Try Problems 13, 17, and 23

2 Write Complex Numbers in $a + bi$ Form.

The imaginary number i is used to define *complex numbers.*

Complex Numbers

A **complex number** is any number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.

Complex numbers of the form $a + bi$, where $b \neq 0$, are also called **imaginary numbers.** Complex numbers of the form bi are also called **pure imaginary numbers.**

For a complex number written in the standard form $a + bi$, we call the real number a the **real part** and the real number b the **imaginary part.**

EXAMPLE 2 Write each number in the form $a + bi$: **a.** 6 **b.** $\sqrt{-16}$ **c.** $-8 + \sqrt{-45}$

Strategy To write each number in the form $a + bi$, we will determine a and bi.

Why The standard form $a + bi$ of a complex number is composed of two parts, the real part a and the imaginary part b.

Solution

a. $6 = 6 + 0i$ The real part is 6 and the imaginary part is 0.

b. To simplify $\sqrt{-16}$, we need to write $\sqrt{-16}$ in terms of i:

$$\sqrt{-16} = \sqrt{-1}\sqrt{16} = 4i$$

Thus, $\sqrt{-16} = 0 + 4i$. The real part is 0 and the imaginary part is 4.

c. To simplify $-8 + \sqrt{-45}$, we need to write $\sqrt{-45}$ in terms of i:

$$\sqrt{-45} = \sqrt{-1}\sqrt{45} = \sqrt{-1}\sqrt{9}\sqrt{5} = 3i\sqrt{5}$$

Thus, $-8 + \sqrt{-45} = -8 + 3i\sqrt{5}$. The real part is -8 and the imaginary part is $3\sqrt{5}$.

Success Tip

Complex numbers are used to describe alternating electric current, they are used to model fluid flow in a pipe, and to represent strains on steel beams in skyscrapers.

Self Check 2 Write each number in the form $a + bi$: **a.** -18 **b.** $\sqrt{-36}$ **c.** $1 + \sqrt{-24}$

Now Try Problems 25 and 29

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The following table shows the relationship between the real numbers, the imaginary numbers, and the complex numbers.

Complex numbers

Real numbers				Imaginary numbers		
-6	$\frac{5}{16}$	-1.75	π	$9 + 7i$	$-2i$	$\frac{1}{4} - \frac{3}{4}i$
48	0	$-\sqrt{10}$	$-\frac{7}{2}$	$0.56i$	$6 + i\sqrt{3}$	

3 Add and Subtract Complex Numbers.

Adding and subtracting complex numbers is similar to adding and subtracting polynomials.

Addition and Subtraction of Complex Numbers

1. To add complex numbers, add their real parts and add their imaginary parts.
2. To subtract complex numbers, add the opposite of the complex number being subtracted.

EXAMPLE 3 Find each sum or difference. Write each result in the form $a + bi$.

a. $(5 + 2i) + (1 + 8i)$ **b.** $(-6 - 5i) - (3 - 4i)$ **c.** $11i + (-2 + 6i)$

Strategy To add the complex numbers, we will add their real parts and add their imaginary parts. To subtract the complex numbers, we will add the opposite of the complex number to be subtracted.

Why We perform the indicated operations as if the complex numbers were polynomials with i as a variable.

Solution

Success Tip

i is not a variable, but it is helpful to think of it as one when adding, subtracting, and multiplying. For example:

$$4i + 3i = 7i$$
$$8i - 6i = 2i$$
$$i \cdot i = i^2$$

a. $(5 + 2i) + (1 + 8i) = (5 + 1) + (2 + 8)i$ The sum of the imaginary parts. The sum of the real parts.

$= 6 + 10i$

b. $(-6 - 5i) - (3 - 4i) = (-6 - 5i) + (-3 + 4i)$ Add the opposite. To find the opposite, change the sign of each term of $3 - 4i$.

$= [-6 + (-3)] + (-5 + 4)i$ Add the real parts. Add the imaginary parts.

$= -9 - i$

c. $11i + (-2 + 6i) = -2 + (11 + 6)i$ Add the imaginary parts.

$= -2 + 17i$

Self Check 3 Find the sum or difference. Write each result in the form $a + bi$:

a. $(3 - 5i) + (-2 + 6i)$

b. $(-4 - i) - (-1 - 6i)$ **c.** $9 + (16 - 4i)$

Now Try Problems 33 and 35

4 Multiply and Divide Complex Numbers.

Multiplying complex numbers is similar to multiplying polynomials.

EXAMPLE 4 Find each product. Write each result in the form $a + bi$. **a.** $5i(4 - i)$ **b.** $(2 + 5i)(6 + 4i)$ **c.** $(3 + 5i)(3 - 5i)$

Strategy We will use the distributive property or the FOIL method to find the products.

Why We perform the indicated operations as if the complex numbers were polynomials with i as a variable.

Solution

a.

$$\begin{aligned} 5i(4 - i) &= 5i \cdot 4 - 5i \cdot i && \text{Distribute the multiplication by } 5i. \\ &= 20i - 5i^2 \\ &= 20i - 5(-1) && \text{Replace } i^2 \text{ with } -1. \\ &= 20i + 5 \\ &= 5 + 20i && \text{Write the result in the form } a + bi. \end{aligned}$$

b.

$$\begin{aligned} (2 + 5i)(6 + 4i) &= \overset{F}{12} + \overset{O}{8i} + \overset{I}{30i} + \overset{L}{20i^2} && \text{Use the FOIL method.} \\ &= 12 + 38i + 20(-1) && 8i + 30i = 38i. \text{ Replace } i^2 \text{ with } -1. \\ &= 12 + 38i - 20 \\ &= -8 + 38i && \text{Subtract: } 12 - 20 = -8. \end{aligned}$$

c.

$$\begin{aligned} (3 + 5i)(3 - 5i) &= \overset{F}{9} - \overset{O}{15i} + \overset{I}{15i} - \overset{L}{25i^2} && \text{Use the FOIL method.} \\ &= 9 - 25(-1) && \text{Add: } -15i + 15i = 0. \text{ Replace } i^2 \text{ with } -1. \\ &= 9 + 25 \\ &= 34 \end{aligned}$$

Written in the form $a + bi$, the product is $34 + 0i$.

Self Check 4 Find the product. Write each result in the form $a + bi$:
a. $4i(3 - 9i)$ **b.** $(3 - 2i)(5 - 4i)$
c. $(2 + 6i)(2 - 6i)$

Now Try Problems 43 and 45

In Example 4c, we saw that the product of the imaginary numbers $3 + 5i$ and $3 - 5i$ is the real number 34. We call $3 + 5i$ and $3 - 5i$ *complex conjugates* of each other.

Complex Conjugates The complex numbers $a + bi$ and $a - bi$ are called **complex conjugates.**

For example,

- $7 + 4i$ and $7 - 4i$ are complex conjugates.
- $5 - i$ and $5 + i$ are complex conjugates.
- $-6i$ and $6i$ are complex conjugates, because $-6i = 0 - 6i$ and $6i = 0 + 6i$.

Success Tip

To find the conjugate of a complex number, write it in standard form and change the sign between the real and imaginary parts from + to − or from − to +.

In general, the product of the complex number $a + bi$ and its complex conjugate $a - bi$ is the real number $a^2 + b^2$, as the following work shows:

$$\begin{aligned} (a + bi)(a - bi) &= \overset{F}{a^2} - \overset{O}{abi} + \overset{I}{abi} - \overset{L}{b^2i^2} && \text{Use the FOIL method.} \\ &= a^2 - b^2(-1) && \text{Add: } -abi + abi = 0. \text{ Replace } i^2 \text{ with } -1. \\ &= a^2 + b^2 \end{aligned}$$

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We can use this fact when dividing by a complex number. The process that we use is similar to rationalizing denominators.

EXAMPLE 5 Write each quotient in the form $a + bi$: **a.** $\frac{6}{7 - 4i}$ **b.** $\frac{3 - i}{2 + i}$

Strategy We will build each fraction by multiplying it by a form of 1 that uses the conjugate of the denominator.

Why This step produces a real number in the denominator so that the result can then be written in the form $a + bi$.

Solution

a. We want to build a fraction equivalent to $\frac{6}{7 - 4i}$ that does not have i in the denominator. To make the denominator, $7 - 4i$, a real number, we need to multiply it by its complex conjugate, $7 + 4i$. It follows that $\frac{7 + 4i}{7 + 4i}$ should be the form of 1 that is used to build $\frac{6}{7 - 4i}$.

$$\frac{6}{7 - 4i} = \frac{6}{7 - 4i} \cdot \frac{7 + 4i}{7 + 4i}$$ To build an equivalent fraction, multiply by $\frac{7 + 4i}{7 + 4i} = 1$.

$$= \frac{42 + 24i}{49 - 16i^2}$$ To multiply the numerators, distribute the multiplication by 6. To multiply the denominators, find $(7 - 4i)(7 + 4i)$.

$$= \frac{42 + 24i}{49 - 16(-1)}$$ Replace i^2 with -1. The denominator no longer contains i.

$$= \frac{42 + 24i}{49 + 16}$$ Simplify the denominator.

$$= \frac{42 + 24i}{65}$$ This notation represents the sum of two fractions that have the common denominator 65: $\frac{42}{65}$ and $\frac{24i}{65}$.

$$= \frac{42}{65} + \frac{24}{65}i$$ Write the result in the form $a + bi$.

b. We can make the denominator of $\frac{3 - i}{2 + i}$ a real number by multiplying it by the complex conjugate of $2 + i$, which is $2 - i$.

$$\frac{3 - i}{2 + i} = \frac{3 - i}{2 + i} \cdot \frac{2 - i}{2 - i}$$ To build an equivalent fraction, multiply by $\frac{2 - i}{2 - i} = 1$.

$$= \frac{6 - 3i - 2i + i^2}{4 - i^2}$$ To multiply the numerators, find $(3 - i)(2 - i)$. To multiply the denominators, find $(2 + i)(2 - i)$.

$$= \frac{6 - 5i + (-1)}{4 - (-1)}$$ Replace i^2 with -1. The denominator no longer contains i.

$$= \frac{5 - 5i}{5}$$ Simplify the numerator and the denominator.

$$= \frac{5}{5} - \frac{5i}{5}$$ Write each term of the numerator over the denominator, 5.

$$= 1 - i$$ Simplify each fraction.

The Language of Algebra

Recall that the word *conjugate* was used in Chapter 8 when we rationalized the denominators of radical expressions such as $\frac{5}{\sqrt{6} - 1}$.

Caution

A common mistake is to replace i with -1. Remember, $i \neq -1$. By definition, $i = \sqrt{-1}$ and $i^2 = -1$.

Notation

It is acceptable to use $a - bi$ as a substitute for the form $a + (-b)i$. In Example 5b, we write $1 - i$ instead of $1 + (-1)i$.

Self Check 5 Write each quotient in standard form: **a.** $\frac{5}{4 - i}$ **b.** $\frac{5 - 3i}{4 + 2i}$

Now Try Problems 49 and 53

As the results of Example 5 show, we use the following rule to divide complex numbers.

Division of Complex Numbers	To divide complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

5 Solve Quadratic Equations with Complex-Number Solutions.

We have seen that certain quadratic equations do not have real-number solutions. In the complex number system, all quadratic equations have solutions. We can write their solutions in the form $a + bi$.

EXAMPLE 6 Solve each equation. Express the solutions in the form $a + bi$: **a.** $x^2 + 25 = 0$ **b.** $(y - 3)^2 = -54$

Strategy We will use the square root property to solve each equation.

Why It is the fastest way to solve equations of this form.

Solution

a. $x^2 + 25 = 0$ — This is the equation to solve.

$x^2 = -25$ — To isolate x^2, subtract 25 from both sides.

$x = \pm\sqrt{-25}$ — Use the square root property. Note that the radicand is negative.

$x = \pm 5i$ — Write $\sqrt{-25}$ in terms of i: $\sqrt{-25} = \sqrt{-1}\sqrt{25} = 5i$.

To express the solutions in the form $a + bi$, we must write 0 for the real part, a. Thus, the solutions are $0 + 5i$ and $0 - 5i$, or more simply, $0 \pm 5i$.

b. $(y - 3)^2 = -54$ — This is the equation to solve.

$y - 3 = \pm\sqrt{-54}$ — Use the square root property. Note that the radicand is negative.

$y - 3 = \pm 3i\sqrt{6}$ — Write $\sqrt{-54}$ in terms of i: $\sqrt{-54} = \sqrt{-1}\sqrt{9}\sqrt{6} = 3i\sqrt{6}$.

$y = 3 \pm 3i\sqrt{6}$ — To isolate y, add 3 to both sides.

The solutions are $3 + 3i\sqrt{6}$ and $3 - 3i\sqrt{6}$, or more simply, $3 \pm 3i\sqrt{6}$.

Self Check 6 Solve each equation. Express the solutions in the form $a + bi$: **a.** $x^2 + 100 = 0$ **b.** $(d - 3)^2 = -48$

Now Try Problems 57 and 63

EXAMPLE 7 Solve $4t^2 - 6t + 3 = 0$. Express the solutions in the form $a + bi$.

Strategy We use the quadratic formula to solve the equation.

Why $4t^2 - 6t + 3$ does not factor.

Solution

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ In the quadratic formula, replace x with t.

$$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(3)}}{2(4)}$$ Substitute 4 for a, -6 for b, and 3 for c.

$$t = \frac{6 \pm \sqrt{36 - 48}}{8}$$ Evaluate the power and multiply within the radical. Multiply in the denominator.

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$$t = \frac{6 \pm \sqrt{-12}}{8}$$ Subtract within the radical: 36 − 48. Note that the radicand is negative.

$$t = \frac{6 \pm 2i\sqrt{3}}{8}$$ Write $\sqrt{-12}$ in terms of i: $\sqrt{-12} = \sqrt{-1}\sqrt{12} = \sqrt{-1}\sqrt{4}\sqrt{3} = 2i\sqrt{3}$

$$t = \frac{\overset{1}{\cancel{2}}(3 \pm i\sqrt{3})}{\underset{1}{\cancel{2}} \cdot 4}$$ Factor out the GCF 2 from $6 \pm 2i\sqrt{3}$ and factor 8 as $2 \cdot 4$. Then remove the common factor 2: $\frac{2}{2} = 1$.

$$t = \frac{3 \pm i\sqrt{3}}{4}$$ This notation represents the sum and difference of two fractions that have the common denominator 4: $\frac{3}{4}$ and $\frac{i\sqrt{3}}{4}$.

Writing each result as a complex number in the form $a + bi$, the solutions are

$$\frac{3}{4} + \frac{\sqrt{3}}{4}i \quad \text{and} \quad \frac{3}{4} - \frac{\sqrt{3}}{4}i \quad \text{or more simply} \quad \frac{3}{4} \pm \frac{\sqrt{3}}{4}i$$

Self Check 7 Solve $a^2 + 2a + 3 = 0$. Express the solutions in the form $a + bi$.

Now Try Problem 65

SECTION 9.4 STUDY SET

VOCABULARY

Fill in the blanks.

1. $9 + 2i$ is an example of a _________ number. The _____ part is 9 and the __________ part is 2.
2. The __________ number i is used to define complex numbers.

CONCEPTS

Fill in the blanks.

3. a. $i =$ ☐ b. $i \cdot i = i^{☐} =$ ☐
4. $\sqrt{-25} = \sqrt{☐ \cdot 25} = ☐\sqrt{25} = 5☐$
5. a. $5i + 3i =$ ☐ b. $5i - 3i =$ ☐
6. The product of any complex number and its complex conjugate is a _____ number.
7. To write the quotient $\frac{2 - 3i}{6 - i}$ as a complex number in standard form, we multiply it by ☐.
8. $\frac{3 \pm \sqrt{-4}}{5} = \frac{3 \pm ☐}{5}$
9. Determine whether each statement is true or false.
   a. Every real number is a complex number.
   b. $2 + 7i$ is an imaginary number.
   c. $\sqrt{-16}$ is a real number.
   d. In the complex number system, all quadratic equations have solutions.
10. Give the complex conjugate of each number.
    a. $2 - 9i$ b. $-8 + i$
    c. $4i$ d. $-11i$

NOTATION

11. Write each expression so it is clear that i is not within the radical symbol.
    a. $\sqrt{7i}$ b. $2\sqrt{3i}$
12. Write $\frac{3 - 4i}{5}$ in the form $a + bi$.

GUIDED PRACTICE

***Write each expression in terms of i.* See Example 1.**

13. $\sqrt{-9}$
14. $\sqrt{-4}$
15. $\sqrt{-7}$
16. $\sqrt{-11}$
17. $\sqrt{-24}$
18. $\sqrt{-28}$
19. $-\sqrt{-32}$
20. $-\sqrt{-72}$
21. $5\sqrt{-81}$
22. $6\sqrt{-49}$
23. $\sqrt{-\frac{25}{9}}$
24. $\sqrt{-\frac{121}{144}}$

***Write each number in the form a + bi.* See Example 2.**

25. 12
26. −27
27. $\sqrt{-100}$
28. $\sqrt{-64}$
29. $6 + \sqrt{-16}$
30. $14 + \sqrt{-25}$
31. $-9 - \sqrt{-49}$
32. $-45 - \sqrt{-36}$

Perform the operations. Write all answers in the form a + bi. See Example 3.

33. $(3 + 4i) + (5 - 6i)$

34. $(5 + 3i) - (6 - 9i)$

35. $(7 - 3i) - (4 + 2i)$

36. $(8 + 3i) + (-7 - 2i)$

37. $(14 - 4i) - 9i$

38. $(20 - 5i) - 17i$

39. $15 + (-3 - 9i)$

40. $-25 + (18 - 9i)$

Perform the operations. Write all answers in the form a + bi. See Example 4.

41. $3(2 - i)$

42. $9(-4 - 4i)$

43. $-5i(5 - 5i)$

44. $2i(7 + 2i)$

45. $(3 - 2i)(2 + 3i)$

46. $(3 - i)(2 + 3i)$

47. $(4 + i)(3 - i)$

48. $(1 - 5i)(1 - 4i)$

Write each quotient in the form a + bi. See Example 5.

49. $\frac{5}{2 - i}$

50. $\frac{26}{3 - 2i}$

51. $\frac{-4i}{7 - 2i}$

52. $\frac{5i}{6 + 2i}$

53. $\frac{2 + 3i}{2 - 3i}$

54. $\frac{2 - 5i}{2 + 5i}$

55. $\frac{4 - 3i}{7 - i}$

56. $\frac{4 + i}{4 - i}$

Solve each equation. Write all solutions in the form a + bi. See Example 6.

57. $x^2 + 9 = 0$

58. $x^2 + 100 = 0$

59. $d^2 + 8 = 0$

60. $a^2 + 27 = 0$

61. $(x + 3)^2 = -1$

62. $(x + 2)^2 = -25$

63. $(x - 11)^2 = -75$

64. $(x - 22)^2 = -18$

Solve each equation. Write all solutions in the form a + bi. See Example 7.

65. $x^2 - 3x + 4 = 0$

66. $y^2 + y + 3 = 0$

67. $2x^2 + x + 1 = 0$

68. $2x^2 + 3x + 3 = 0$

TRY IT YOURSELF

Perform the operations. Write all answers in the form a + bi.

69. $\frac{3}{5 + i}$

70. $\frac{-4}{7 - 2i}$

71. $(6 - i) + (9 + 3i)$

72. $(5 - 4i) + (3 + 2i)$

73. $(-3 - 8i) - (-3 - 9i)$

74. $(-1 - 8i) - (-1 - 7i)$

75. $(2 + i)(2 + 3i)$

76. $2i(7 + 2i)$

77. $\frac{3 - 2i}{3 + 2i}$

78. $\frac{3 + 2i}{3 + i}$

79. $(10 - 9i) + (-1 + i)$

80. $(32 - 3i) + (-44 + 15i)$

81. $-4(3 + 4i)$

82. $-7(5 - 3i)$

83. $(2 + i)^2$

84. $(3 - 2i)^2$

Solve each equation. Write all solutions in the form a + bi.

85. $2x^2 + x = -5$

86. $4x^2 = -7x - 4$

87. $(x - 4)^2 = -45$

88. $(x - 9)^2 = -80$

89. $b^2 + 2b + 2 = 0$

90. $t^2 - 2t + 6 = 0$

91. $x^2 = -36$

92. $x^2 = -49$

93. $x^2 = -\frac{16}{9}$

94. $x^2 = -\frac{25}{4}$

95. $3x^2 + 2x + 1 = 0$

96. $3x^2 - 4x + 2 = 0$

APPLICATIONS

97. Electricity. In an AC (alternating current) circuit, if two sections are connected in series and have the same current in each section, the voltage is given by $V = V_1 + V_2$. Find the total voltage in a given circuit if the voltages in the individual sections are $V_1 = 10.31 - 5.97i$ and $V_2 = 8.14 + 3.79i$.

98. Electronics. The impedance Z in an AC (alternating current) circuit is a measure of how much the circuit impedes (hinders) the flow of current through it. The impedance is related to the voltage V and the current I by the formula

$$V = IZ$$

If a circuit has a current of $(0.5 + 2.0i)$ amps and an impedance of $(0.4 - 3.0i)$ ohms, find the voltage.

WRITING

99. What unusual situation discussed at the beginning of this section illustrated the need to define the square root of a negative number?

100. Explain the difference between the opposite of a complex number and its conjugate.

101. What is an imaginary number?

102. In this section, we have seen that $i^2 = -1$. From your previous experience in this course, what is unusual about that fact?

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REVIEW

Rationalize the denominator.

103. $\dfrac{1}{\sqrt{7}}$

104. $\dfrac{\sqrt{3}}{\sqrt{10}}$

105. $\dfrac{8}{\sqrt{x} - 2}$

106. $\dfrac{\sqrt{3}}{5 + \sqrt{x}}$

CHALLENGE PROBLEMS

Perform the operations.

107. $(2\sqrt{2} + i\sqrt{2})(3\sqrt{2} - i\sqrt{2})$

108. $\dfrac{\sqrt{5} - i\sqrt{3}}{\sqrt{5} + i\sqrt{3}}$

SECTION 9.5 Graphing Quadratic Equations

OBJECTIVES

1. Understand the vocabulary used to describe parabolas.
2. Find the intercepts of a parabola.
3. Determine the vertex of a parabola.
4. Graph equations of the form $y = ax^2 + bx + c$.
5. Solve quadratic equations graphically.

ARE YOU READY?

The following problems review some basic skills that are needed when graphing quadratic equations.

1. Solve: $x^2 - 2x - 3 = 0$
2. Evaluate $x^2 + 3x - 8$ for $x = -1$.
3. What is the coefficient of each term of the expression $x^2 - 4x + 6$?
4. Approximate $1 + \sqrt{3}$ to the nearest tenth.

In this section, we will combine our graphing skills with our equation-solving skills to graph *quadratic equations in two variables.*

1 Understand the Vocabulary Used to Describe Parabolas.

Equations that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$, are called **quadratic equations in two variables.** Some examples are

$$y = x^2 - 2x - 3 \qquad y = -2x^2 - 8x - 8 \qquad y = x^2 + x$$

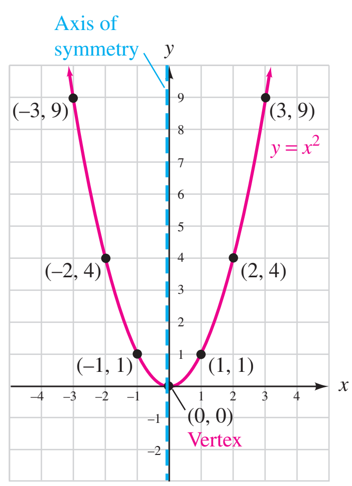
In Section 5.4, we graphed $y = x^2$, a quadratic equation in two variables. To do this, we constructed a table of solutions, plotted points, and joined them with a smooth curve, called a **parabola.** The parabola opens upward, and the lowest point on the graph, called the **vertex,** is the point (0, 0). If we fold the graph paper along the y-axis, the two sides of the parabola match. We say that the graph is *symmetric about the y-axis* and we call the y-axis the **axis of symmetry.**

The Language of Algebra

An **axis of symmetry** (or **line of symmetry**) divides a parabola into two matching sides. The sides are said to be **mirror images** of each other.

$y = x^2$

x	y	(x, y)
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)



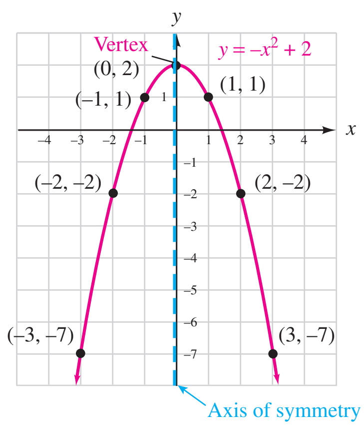
In Section 5.4, we also graphed $y = -x^2 + 2$, another quadratic equation in two variables. The resulting parabola opens downward, and the **vertex** (in this case, the highest point on the graph) is the point (0, 2). The axis of symmetry is the y-axis.

The Language of Algebra

In the equation $y = ax^2 + bx + c$, each value of x determines exactly one value of y. Therefore, the equation defines y to be a **function** of x and we could write $f(x) = ax^2 + bx + c$. Your instructor may ask you to use the vocabulary and notation of functions throughout this section.

$y = -x^2 + 2$

x	y	(x, y)
−3	−7	(−3, −7)
−2	−2	(−2, −2)
−1	1	(−1, 1)
0	2	(0, 2)
1	1	(1, 1)
2	−2	(2, −2)
3	−7	(3, −7)



For the equation $y = x^2$, the coefficient of the x^2 term is the positive number 1. For $y = -x^2 + 2$, the coefficient of the x^2 term is the negative number −1. These observations suggest the following fact.

Graphs of Quadratic Equations

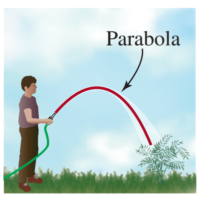
The graph of $y = ax^2 + bx + c$, where $a \neq 0$, is a parabola. It opens upward when $a > 0$ and downward when $a < 0$.

The Language of Algebra

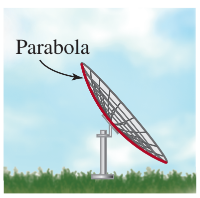
The word **parabolic** (pronounced *par • a • BOL • ic*) means having the form of a parabola. For example, the light bulb in most flashlights is surrounded by a *parabolic* reflecting mirror.

Parabolic shapes can be seen in a wide variety of real-world settings. These shapes can be modeled by quadratic equations in two variables.

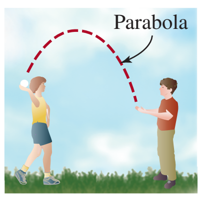
The path of a stream of water



The shape of a satellite antenna dish



The path of a thrown object



2 Find the Intercepts of a Parabola.

When graphing quadratic equations, it is helpful to know the x- and y-intercepts of the parabola. To find the intercepts of a parabola, we use the same steps that we used to find the intercepts of the graphs of linear equations.

Finding Intercepts

To find the y-intercept, substitute 0 for x in the given equation and solve for y.

To find the x-intercepts, substitute 0 for y in the given equation and solve for x.

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EXAMPLE 1 Find the y- and x-intercepts of the graph of $y = x^2 - 2x - 3$.

Strategy To find the y-intercept of the graph, we will let $x = 0$ and find y. To find the x-intercepts of the graph, we will let $y = 0$ and solve the resulting equation for x.

Why A point on the y-axis has an x-coordinate of 0. A point on the x-axis has a y-coordinate of 0.

Solution We let $x = 0$ and evaluate the right side to find the y-intercept.

$y = x^2 - 2x - 3$ This is the given equation.

$y = 0^2 - 2(0) - 3$ Substitute 0 for x.

$y = -3$ Evaluate the right side.

The y-intercept is $(0, -3)$. We note that the y-coordinate of the y-intercept is the same as the value of the constant term c on the right side of $y = x^2 - 2x - 3$.

Next, we let $y = 0$ and solve the resulting quadratic equation to find the x-intercepts.

$y = x^2 - 2x - 3$ This is the given equation.

$0 = x^2 - 2x - 3$ Substitute 0 for y.

$0 = (x - 3)(x + 1)$ Factor the trinomial.

$x - 3 = 0$ or $x + 1 = 0$ Set each factor equal to 0.

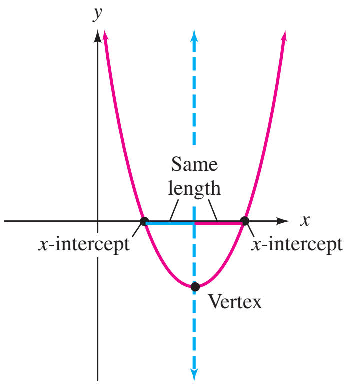
$x = 3$ | $x = -1$

Since there are two solutions, the graph has two x-intercepts: $(3, 0)$ and $(-1, 0)$.

Self Check 1 Find the y- and x-intercepts of the graph of $y = x^2 + 6x + 8$.

Now Try Problem 17

3 Determine the Vertex of a Parabola.



It is usually easier to graph a quadratic equation when we know the coordinates of the vertex of its graph. Because of symmetry, if a parabola has two x-intercepts, the x-coordinate of the vertex is exactly midway between them. We can use this fact to derive a formula to find the vertex of a parabola.

In general, if a parabola has two x-intercepts, they can be found by solving $0 = ax^2 + bx + c$ for x. We can use the quadratic formula to find the solutions. They are

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Thus, the parabola's x-intercepts are $\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}, 0\right)$ and $\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

Since the x-value of the vertex of a parabola is halfway between the two x-intercepts, we can find this value by finding the average, or $\frac{1}{2}$ of the sum of the x-coordinates of the x-intercepts.

$$x = \frac{1}{2}\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)$$

$$x = \frac{1}{2}\left(\frac{-b - \sqrt{b^2 - 4ac} + (-b) + \sqrt{b^2 - 4ac}}{2a}\right)$$ Add the numerators and keep the common denominator.

$$x = \frac{1}{2}\left(\frac{-2b}{2a}\right)$$ Simplify: $-b + (-b) = -2b$ and $-\sqrt{b^2 - 4ac} + \sqrt{b^2 - 4ac} = 0$.

$$x = \frac{-b}{2a}$$ Simplify the fraction within the parentheses.

This result is true even if the graph has no x-intercepts.

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Finding the Vertex of a Parabola

The graph of the quadratic equation $y = ax^2 + bx + c$ is a parabola whose vertex has an x-coordinate of $\frac{-b}{2a}$. To find the y-coordinate of the vertex, substitute $\frac{-b}{2a}$ for x into the equation and find y.

EXAMPLE 2 Find the vertex of the graph of $y = x^2 - 2x - 3$.

Strategy We will compare the given equation to the general form $y = ax^2 + bx + c$ to identify a and b.

Why To use the vertex formula, we need to know a and b.

Solution From the following diagram, we see that $a = 1$, $b = -2$, and $c = -3$.

$$y = 1x^2 - 2x - 3$$
$$\uparrow \quad \uparrow \quad \uparrow$$
$$y = ax^2 + bx + c$$

To find the x-coordinate of the vertex, we substitute the values for a and b into the formula $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$$

The x-coordinate of the vertex is 1. To find the y-coordinate, we substitute 1 for x in the original equation.

$$y = x^2 - 2x - 3 \quad \text{This is the given equation.}$$
$$y = 1^2 - 2(1) - 3 \quad \text{Substitute 1 for x.}$$
$$y = -4 \quad \text{Evaluate the right side.}$$

The vertex of the parabola is $(1, -4)$.

Success Tip

An easy way to remember the vertex formula is to note that $x = \frac{-b}{2a}$ is part of the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Self Check 2 Find the vertex of the graph of $y = x^2 + 6x + 8$.

Now Try Problem 21

4 Graph Equations of the Form $y = ax^2 + bx + c$.

Much can be determined about the graph of $y = ax^2 + bx + c$ from the coefficients a, b, and c. We can use this information to help graph the equation.

Graphing a Quadratic Equation $y = ax^2 + bx + c$

1. *Test for opening upward/downward* Determine whether the parabola opens upward or downward. If $a > 0$, the graph opens upward. If $a < 0$, the graph opens downward.
2. *Find the vertex/axis of symmetry* The x-coordinate of the vertex of the parabola is $x = \frac{-b}{2a}$. To find the y-coordinate, substitute $\frac{-b}{2a}$ for x into the equation and find y. The axis of symmetry is the vertical line passing through the vertex.
3. *Find the intercepts* To find the y-intercept, substitute 0 for x in the given equation and solve for y. The result will be c. Thus, the y-intercept is $(0, c)$.

   To find the x-intercepts (if any), substitute 0 for y and solve the resulting quadratic equation $ax^2 + bx + c = 0$. If no real-number solutions exist, the graph has no x-intercepts.
4. *Plotting points/using symmetry* To find two more points on the graph, select a convenient value for x and find the corresponding value of y. Plot that point and its mirror image on the opposite side of the axis of symmetry.
5. *Draw the parabola* Draw a smooth curve through the located points.

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EXAMPLE 3 Graph: $y = x^2 - 2x - 3$

Strategy We will use the five-step procedure described on the previous page to sketch the graph of the equation.

Why This strategy is usually faster than making a table of solutions and plotting points.

Solution

Success Tip

When graphing, remember that any point on a parabola to the right of the axis of symmetry yields a second point to the left of the axis of symmetry, and vice versa. Think of it as two-for-one.

Upward/downward: The equation is in the form $y = ax^2 + bx + c$, with $a = 1$, $b = -2$, and $c = -3$. Since $a > 0$, the parabola opens upward.

Vertex/axis of symmetry: In Example 2, we found that the vertex of the graph is $(1, -4)$. The axis of symmetry will be the vertical line passing through $(1, -4)$. See part (a) of the figure below.

Intercepts: In Example 1, we found that the y-intercept is $(0, -3)$ and x-intercepts are $(3, 0)$ and $(-1, 0)$.

If the point $(0, -3)$, which is 1 unit to the left of the axis of symmetry, is on the graph, the point $(2, -3)$, which is 1 unit to the right of the axis of symmetry, is also on the graph. See part (a).

Success Tip

To find additional points on the graph, select values of x that are close to the x-coordinate of the vertex.

Plotting points/using symmetry: It would be helpful to locate two more points on the graph. To find a solution of $y = x^2 - 2x - 3$, we select a convenient value for x, say -2, and find the corresponding value of y.

$y = x^2 - 2x - 3$ This is the equation to graph.

$y = (-2)^2 - 2(-2) - 3$ Substitute -2 for x.

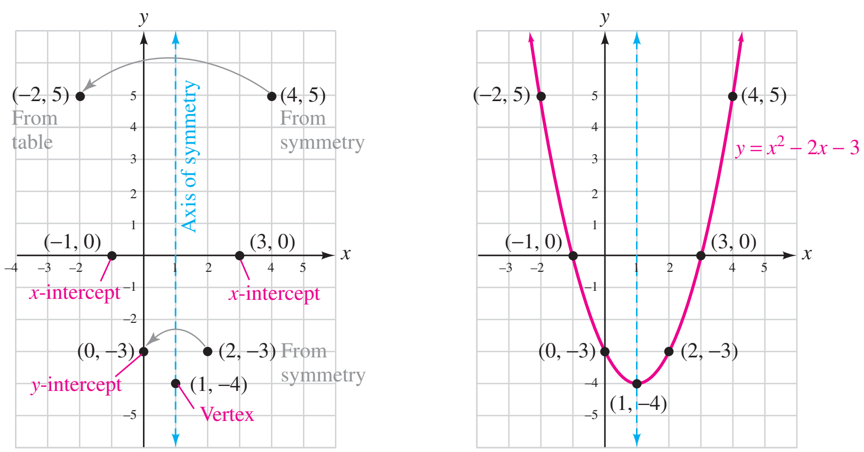
$y = 5$ Evaluate the right side.

$y = x^2 - 2x - 3$

x	y	(x, y)
-2	5	$(-2, 5)$

Thus, the point $(-2, 5)$ lies on the parabola. If the point $(-2, 5)$, which is 3 units to the left of the axis of symmetry, is on the graph, the point $(4, 5)$, which is 3 units to the right of the axis of symmetry, is also on the graph. See part (a).

Draw a smooth curve through the points: The completed graph of $y = x^2 - 2x - 3$ is shown in part (b).



(a) (b)

Self Check 3 Use your results from Self Checks 1 and 2 to help graph $y = x^2 + 6x + 8$.

Now Try Problem 25

EXAMPLE 4 Graph: $y = -2x^2 - 8x - 8$

Strategy We will follow the five-step procedure to sketch the graph of the equation.

Why This strategy is usually faster than making a table of solutions and plotting points.

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Solution

> **Success Tip**
>
> The most important point to find when graphing a quadratic equation in two variables is the vertex.

Upward/downward: The equation is in the form $y = ax^2 + bx + c$, with $a = -2$, $b = -8$, and $c = -8$. Since $a < 0$, the parabola opens downward.

Vertex/axis of symmetry: To find the x-coordinate of the vertex, we substitute -2 for a and -8 for b into the formula $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = -2$$

The x-coordinate of the vertex is -2. To find the y-coordinate, we substitute -2 for x in the original equation and find y.

$$y = -2x^2 - 8x - 8 \quad \text{This is the equation to graph.}$$
$$y = -2(-2)^2 - 8(-2) - 8 \quad \text{Substitute } -2 \text{ for } x.$$
$$y = 0 \quad \text{Evaluate the right side.}$$

The vertex of the parabola is the point $(-2, 0)$. The axis of symmetry is the vertical line passing through $(-2, 0)$. See part (a) of the figure.

Intercepts: Since $c = -8$, the y-intercept of the parabola is $(0, -8)$. The point $(-4, -8)$, which is 2 units to the left of the axis of symmetry, must also be on the graph. See part (a).

To find the x-intercepts, we let $y = 0$ and solve the resulting quadratic equation.

> **Success Tip**
>
> To solve $0 = -2x^2 - 8x - 8$, instead of dividing both sides by -2, we could factor out the common factor -2 and proceed as follows:
>
> $$0 = -2x^2 - 8x - 8$$
> $$0 = -2(x^2 + 4x + 4)$$
> $$0 = -2(x + 2)(x + 2)$$

$$y = -2x^2 - 8x - 8 \quad \text{This is the equation to graph.}$$
$$0 = -2x^2 - 8x - 8 \quad \text{Substitute 0 for } y.$$
$$0 = x^2 + 4x + 4 \quad \text{Divide both sides by } -2\text{: } \frac{0}{-2} = \frac{-2x^2}{-2} - \frac{8x}{-2} - \frac{8}{-2}.$$
$$0 = (x + 2)(x + 2) \quad \text{Factor the trinomial.}$$
$$x + 2 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{Set each factor equal to 0.}$$
$$x = -2 \qquad\qquad x = -2$$

Since the solutions are the same, the graph has only one x-intercept: $(-2, 0)$. This point is the vertex of the parabola and has already been plotted.

$y = -2x^2 - 8x - 8$

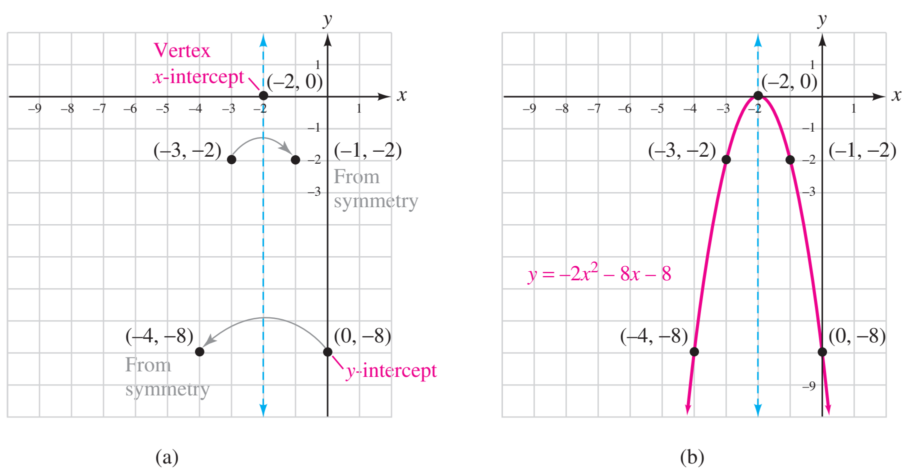
x	y	(x, y)
-3	-2	$(-3, -2)$

Plotting points/using symmetry: It would be helpful to know two more points on the graph. To find a solution of $y = -2x^2 - 8x - 8$, we select a convenient value for x, say -3, and find the corresponding value for y.

$$y = -2x^2 - 8x - 8 \quad \text{This is the equation to graph.}$$
$$y = -2(-3)^2 - 8(-3) - 8 \quad \text{Substitute } -3 \text{ for } x.$$
$$y = -2 \quad \text{Evaluate the right side.}$$

Thus, the point $(-3, -2)$ lies on the parabola. We plot $(-3, -2)$ and then use symmetry to determine that $(-1, -2)$ is also on the graph. See part (a).

Draw a smooth curve through the points: The completed graph of $y = -2x^2 - 8x - 8$ is shown in part (b).



(a) (b)

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Self Check 4 Graph: $y = -2x^2 + 4x - 2$

Now Try Problem 29

EXAMPLE 5 Graph: $y = x^2 + 2x - 2$

Strategy We will follow the five-step procedure to sketch the graph of the equation.

Why This strategy is usually faster than making a table of solutions and plotting points.

Solution ***Upward/downward:*** The equation is in the form $y = ax^2 + bx + c$, with $a = 1$, $b = 2$, and $c = -2$. Since $a > 0$, the parabola opens upward.

Vertex/axis of symmetry: To find the x-coordinate of the vertex, we substitute 1 for a and 2 for b into the formula $x = \frac{-b}{2a}$.

$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

The x-coordinate of the vertex is -1. To find the y-coordinate, we substitute -1 for x in the original equation and find y.

$$y = x^2 + 2x - 2 \quad \text{This is the equation to graph.}$$
$$y = (-1)^2 + 2(-1) - 2 \quad \text{Substitute } -1 \text{ for } x.$$
$$y = -3 \quad \text{Evaluate the right side.}$$

The vertex of the parabola is the point $(-1, -3)$. The axis of symmetry is the vertical line passing through $(-1, -3)$. See part (a) of the figure on the next page.

Intercepts: Since $c = -2$, the y-intercept of the parabola is $(0, -2)$. The point $(-2, -2)$, which is one unit to the left of the axis of symmetry, must also be on the graph. See part (a).

To find the x-intercepts, we let $y = 0$ and solve the resulting quadratic equation.

$$y = x^2 + 2x - 2 \quad \text{This is the equation to graph.}$$
$$0 = x^2 + 2x - 2 \quad \text{Substitute 0 for } y.$$

Since $x^2 + 2x - 2$ does not factor, we will use the quadratic formula to solve for x.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-2)}}{2(1)} \quad \text{In the quadratic formula, substitute 1 for } a\text{, 2 for } b\text{, and } -2 \text{ for } c.$$

$$x = \frac{-2 \pm \sqrt{12}}{2} \quad \text{Evaluate the right side.}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} \quad \text{Simplify the radical: } \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}.$$

$$x = \frac{\overset{1}{\cancel{2}}(-1 \pm \sqrt{3})}{\underset{1}{\cancel{2}}} \quad \text{Factor out the GCF, 2, from the terms in the numerator. Then simplify by removing the common factor of 2 in the numerator and denominator.}$$

$$x = -1 \pm \sqrt{3}$$

The x-intercepts of the graph are $\left(-1 + \sqrt{3}, 0\right)$ and $\left(-1 - \sqrt{3}, 0\right)$. To help to locate their positions on the graph, we can use a calculator to approximate the two irrational numbers. See part (a) on the next page.

$$-1 + \sqrt{3} \approx 0.7 \qquad \text{and} \qquad -1 - \sqrt{3} \approx -2.7$$

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$y = x^2 + 2x - 2$

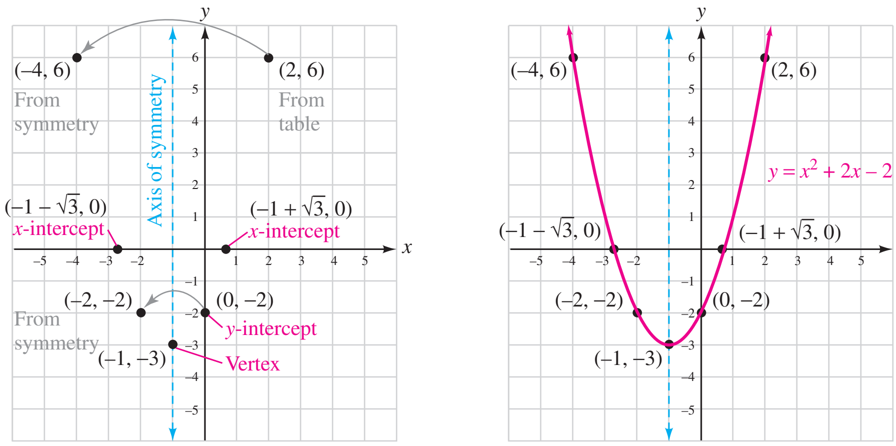
x	y	(x, y)
2	6	(2, 6)

Success Tip

To plot the x-intercepts, we use the approximations $(0.7, 0)$ and $(-2.7, 0)$ to locate their position on the x-axis. However, we label the points on the graph using the exact values: $\left(-1 + \sqrt{3}, 0\right)$ and $\left(-1 - \sqrt{3}, 0\right)$.

Plotting points/using symmetry: To find two more points on the graph, we let $x = 2$, substitute 2 for x in $y = x^2 + 2x - 2$, and find that y is 6. We plot (2, 6) and then use symmetry to determine that $(-4, 6)$ is also on the graph. See part (a).

Draw a smooth curve through the points: The completed graph of $y = x^2 + 2x - 2$ is shown in part (b).



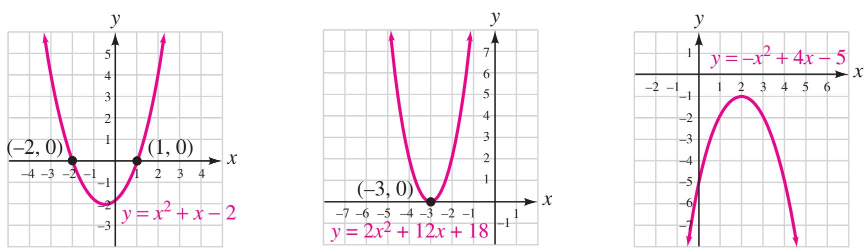
(a) (b)

Self Check 5 Graph: $y = x^2 - 4x - 3$

Now Try Problem 43

5 Solve Quadratic Equations Graphically.

The number of distinct x-intercepts of the graph of $y = ax^2 + bx + c$ is the same as the number of distinct real-number solutions of $ax^2 + bx + c = 0$. For example, the graph of $y = x^2 + x - 2$ in part (a) of the following figure has two x-intercepts, and $x^2 + x - 2 = 0$ has two real-number solutions. In part (b), the graph has one x-intercept, and the corresponding equation has one real-number solution. In part (c), the graph does not have an x-intercept, and the corresponding equation does not have any real-number solutions. Note that the solutions of each equation are given by the x-coordinates of the x-intercepts of each respective graph.



$x^2 + x - 2 = 0$ has two solutions, -2 and 1.
(a)

$2x^2 + 12x + 18 = 0$ has one repeated solution, -3.
(b)

$-x^2 + 4x - 5 = 0$ has no real-number solutions.
(c)

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SECTION 9.5 STUDY SET

VOCABULARY

Fill in the blanks.

1. $y = 3x^2 + 5x - 1$ is a _________ equation in two variables. Its graph is a cup-shaped figure called a _________.
2. The lowest point on a parabola that opens upward, and the highest point on a parabola that opens downward, is called the _______ of the parabola.
3. Points where a parabola intersects the x-axis are called the x-_________ of the graph and the point where a parabola intersects the y-axis is called the y-_________ of the graph.
4. The vertical line that splits the graph of a parabola into two identical parts is called the axis of __________.

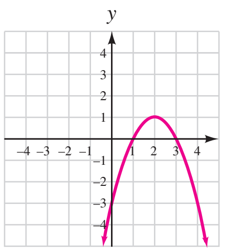
CONCEPTS

Fill in the blanks.

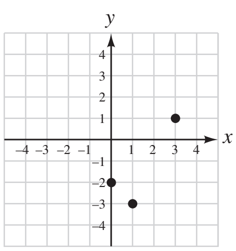
5. The graph of $y = ax^2 + bx + c$ opens downward when a ___ 0 and upward when $a >$ ___.
6. The graph of $y = ax^2 + bx + c$ is a parabola whose vertex has an x-coordinate given by ___.
7. a. To find the y-intercepts of a graph, substitute ___ for x in the given equation and solve for y.
   b. To find the x-intercepts of a graph, substitute 0 for y in the given equation and solve for ___.
8. $y = x^2 - 3x - 1$

x	y	(x, y)
3		(3,)

9. a. What do we call the curve shown in the graph?
   b. What are the x-intercepts of the graph?
   c. What is the y-intercept of the graph?
   d. What is the vertex?
   e. Draw the axis of symmetry on the graph.



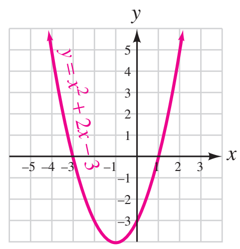
10. Does the graph of each quadratic equation open upward or downward?
    a. $y = 2x^2 + 5x - 1$
    b. $y = -6x^2 - 3x + 5$
11. The vertex of a parabola is $(1, -3)$, its y-intercept is $(0, -2)$, and it passes through the point $(3, 1)$. Draw the axis of symmetry and use it to help determine two other points on the parabola.



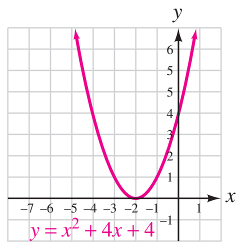
12. Sketch the graph of a quadratic equation using the given facts about its graph.
    - Opens upward
    - Vertex: $(3, -1)$
    - y-intercept: $(0, 8)$
    - x-intercepts: $(2, 0)$, $(4, 0)$

x	y	(x, y)
1	3	(1, 3)

13. Examine the graph of $y = x^2 + 2x - 3$. How many real-number solutions does the equation $x^2 + 2x - 3 = 0$ have? Find them.



14. Examine the graph of $y = x^2 + 4x + 4$. How many real-number solutions does the equation $x^2 + 4x + 4 = 0$ have? Find them.



NOTATION

15. Consider the equation $y = 2x^2 + 4x - 8$.
    a. What are a, b, and c?
    b. Find $\frac{-b}{2a}$.
16. Evaluate: $\frac{-(-12)}{2(-3)}$

GUIDED PRACTICE

Find the x- and y-intercepts of the graph of the quadratic equation. See Example 1.

17. $y = x^2 - 6x + 8$
18. $y = 2x^2 - 4x$
19. $y = -x^2 - 10x - 21$
20. $y = 3x^2 + 6x - 9$

Find the vertex of the graph of each quadratic equation. See Example 2.

21. $y = 2x^2 - 4x + 1$
22. $y = 2x^2 + 8x - 4$
23. $y = -x^2 + 6x - 8$
24. $y = -x^2 - 2x - 1$

Graph each quadratic equation by finding the vertex, the x- and y-intercepts, and the axis of symmetry of its graph. See Examples 3 and 4.

25. $y = x^2 + 2x - 3$
26. $y = x^2 + 6x + 5$
27. $y = 2x^2 + 8x + 6$
28. $y = 3x^2 - 12x + 9$

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29. $y = -x^2 + 2x + 3$
30. $y = -2x^2 + 4x$
31. $y = -x^2 + 5x - 4$
32. $y = -x^2 + 2x - 1$
33. $y = x^2 - 2x$
34. $y = x^2 + x$
35. $y = x^2 + 4x + 4$
36. $y = x^2 - 6x + 9$
37. $y = -x^2 - 4x$
38. $y = -x^2 + 2x$
39. $y = 2x^2 + 3x - 2$
40. $y = 3x^2 - 7x + 2$
41. $y = 4x^2 - 12x + 9$
42. $y = -x^2 - 2x - 1$

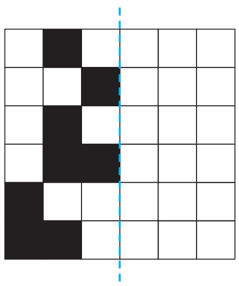
Graph each quadratic equation by finding the vertex, the x- and y-intercepts, and the axis of symmetry of its graph. See Example 5.

43. $y = x^2 - 4x - 1$
44. $y = x^2 + 2x - 5$
45. $y = -x^2 - 2x + 2$
46. $y = -x^2 - 4x + 3$
47. $y = -x^2 - 6x - 4$
48. $y = x^2 - 6x + 4$
49. $y = x^2 - 6x + 10$
50. $y = x^2 - 2x + 4$

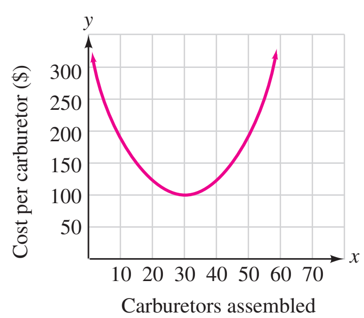
APPLICATIONS

51. Biology. Draw an axis of symmetry over the sketch of the butterfly.

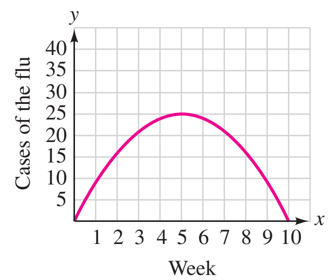
52. Crossword Puzzles. Darken the appropriate squares to the right of the dashed blue line so that the puzzle has symmetry with respect to that line.



53. Cost Analysis. A company has found that when it assembles x carburetors in a production run, the manufacturing cost of $\$y$ per carburetor is given by the graph below. What important piece of information does the vertex give?



54. Health Department. The number of cases of flu seen by doctors at a county health clinic each week during a 10-week period is described by the graph. Write a brief summary report about the flu outbreak. What important piece of information does the vertex give?

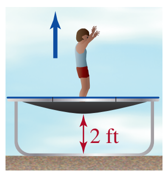


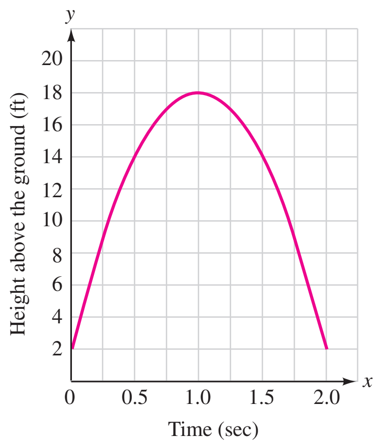
55. Trampolines. The graph shows how far a trampolinist is from the ground (in relation to time) as she bounds into the air and then falls back down to the trampoline.

a. How many feet above the ground is she $\frac{1}{2}$ second after bounding upward?

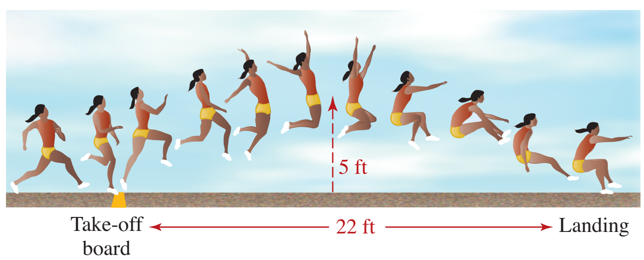
b. When is she 9 feet above the ground?

c. What is the maximum number of feet above the ground she gets? When does this occur?





56. Track and Field. Sketch the parabolic path traveled by the long-jumper's center of gravity from the take-off board to the landing. Let the x-axis represent the ground.



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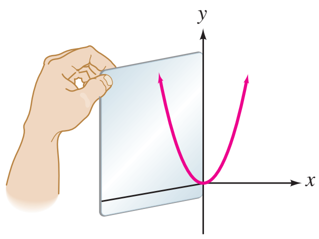
WRITING

57. A mirror is held against the y-axis of the graph of a quadratic equation. What fact about parabolas does this illustrate?



58. Use the example of a stream of water from a drinking fountain to explain the concept of the vertex of a parabola.

59. Explain why the y-intercept of the graph of the quadratic equation $y = ax^2 + bx + c$ is $(0, c)$.

60. Explain how to determine from its equation whether the graph of a parabola opens upward or downward.

61. Is it possible for the graph of a parabola with equation of the form $y = ax^2 + bx + c$ not to have a y-intercept? Explain.

62. Sketch the graphs of parabolas with zero, one, and two x-intercepts. Can the graph of a quadratic equation of the form $y = ax^2 + bx + c$ have more than two x-intercepts? Explain why or why not.

REVIEW

Simplify each expression.

63. $\sqrt{8} - \sqrt{50} + \sqrt{72}$

64. $(4\sqrt{x})(-2\sqrt{x})$

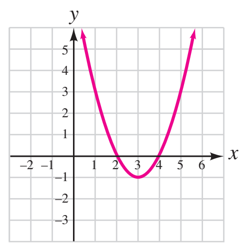
65. $3\sqrt{z}(\sqrt{4z} - \sqrt{z})$

66. $\sqrt[3]{27y^3z^6}$

CHALLENGE PROBLEMS

67. Graph $y = x^2 - x - 2$ and use the graph to solve the quadratic equation $-2 = x^2 - x - 2$.

68. What quadratic equation is graphed here?



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SECTION 9.4 Complex Numbers

DEFINITIONS AND CONCEPTS	EXAMPLES
The **imaginary number** i is defined as $i = \sqrt{-1}$ From the definition, it follows that $i^2 = -1$. A **complex number** is any number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. We call a the **real part** and b the **imaginary part.** If $b \neq 0$, $a + bi$ is also an **imaginary number.**	Write each expression in terms of i: $\sqrt{-100} = \sqrt{-1 \cdot 100} = \sqrt{-1}\sqrt{100} = i \cdot 10 = 10i$ $\sqrt{-75} = \sqrt{-1 \cdot 75} = \sqrt{-1}\sqrt{75} = i\sqrt{25}\sqrt{3} = 5i\sqrt{3}$ or $5\sqrt{3}i$ Show that each number is a complex number by writing it in the form $a + bi$. $10 = 10 + 0i$ — 10 is the real part and 0 is the imaginary part. $5i = 0 + 5i$ — 0 is the real part and 5 is the imaginary part. $-2 - \sqrt{-16} = -2 - 4i$ — -2 is the real part and -4 is the imaginary part.
Adding and subtracting complex numbers is similar to adding and subtracting polynomials. To **add two complex numbers,** add their real parts and add their imaginary parts. To **subtract two complex numbers,** add the opposite of the complex number being subtracted.	Add: $(3 - 4i) + (5 + 7i) = (3 + 5) + (-4 + 7)i$ — Add the real parts. Add the imaginary parts. $= 8 + 3i$ Subtract: $(3 - 4i) - (5 + 7i) = (3 - 4i) + (-5 - 7i)$ — Add the opposite of $5 + 7i$. $= (3 - 5) + [-4 + (-7)]i$ — Add the real parts. Add the imaginary parts. $= -2 - 11i$
Multiplying complex numbers is similar to multiplying polynomials.	Find each product: $7i(4 - 9i) = 28i - 63i^2 = 28i - 63(-1) = 28i + 63 = 63 + 28i$ $(2 + 3i)(4 - 3i) = 8 - 6i + 12i - 9i^2 = 8 + 6i - 9(-1) = 8 + 6i + 9 = 17 + 6i$
The complex numbers $a + bi$ and $a - bi$ are called **complex conjugates.**	The complex numbers $4 - 3i$ and $4 + 3i$ are complex conjugates.
To write the **quotient of two complex numbers** in the form $a + bi$, multiply the numerator and denominator by the complex conjugate of the denominator. The process is similar to rationalizing denominators. This process is similar to rationalizing two-term radical denominators.	Write each quotient in the form $a + bi$: $\frac{6}{4 - 3i} = \frac{6}{4 - 3i} \cdot \frac{4 + 3i}{4 + 3i} = \frac{24 + 18i}{16 - 9i^2} = \frac{24 + 18i}{16 - 9(-1)} = \frac{24 + 18i}{16 + 9} = \frac{24 + 18i}{25} = \frac{24}{25} + \frac{18}{25}i$ $\frac{2 + i}{1 + i} = \frac{2 + i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{2 - 2i + i - i^2}{1 - i^2} = \frac{2 - i - (-1)}{1 - (-1)} = \frac{3 - i}{2} = \frac{3}{2} - \frac{1}{2}i$

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Some quadratic equations have **complex solutions that are imaginary numbers.**

Use the quadratic formula to solve: $p^2 + p + 3 = 0$

Here, $a = 1$, $b = 1$, and $c = 3$.

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ In the quadratic formula, replace *x* with *p*.

$$p = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)}$$ Substitute 1 for *a*, 1 for *b*, and 3 for *c*.

$$p = \frac{-1 \pm \sqrt{-11}}{2}$$ Evaluate the power and multiply within the radical. Multiply in the denominator.

$$p = \frac{-1 \pm i\sqrt{11}}{2}$$ Write $\sqrt{-11}$ in terms of *i*.

$$p = -\frac{1}{2} \pm \frac{\sqrt{11}}{2}i$$ Write the solutions in the form $a + bi$.

REVIEW EXERCISES

Write each expression in terms of i.

47. $\sqrt{-25}$ **48.** $\sqrt{-18}$

49. $-\sqrt{-49}$ **50.** $\sqrt{-\frac{9}{64}}$

51. Complete the diagram.

Complex numbers

52. Determine whether each statement is true or false.
- **a.** Every real number is a complex number.
- **b.** $3 - 4i$ is an imaginary number.
- **c.** $\sqrt{-4}$ is a real number.
- **d.** i is a real number.

Give the complex conjugate of each number.

53. $3 + 6i$ **54.** $-1 - 7i$

55. $19i$ **56.** $-i$

Perform the operations. Write all answers in the form a + bi.

57. $(3 + 4i) + (5 - 6i)$ **58.** $(7 - 3i) - (4 + 2i)$

59. $3i(2 - i)$ **60.** $(2 + 3i)(3 - i)$

61. $\frac{2 + 3i}{2 - 3i}$ **62.** $\frac{3}{5 + i}$

Solve each equation. Write all solutions in the form a + bi.

63. $x^2 + 9 = 0$ **64.** $3x^2 = -16$

65. $(p - 2)^2 = -24$ **66.** $(q + 3)^2 = -54$

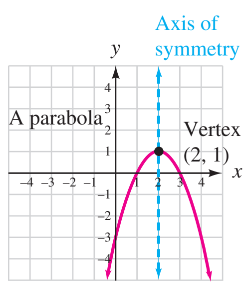
67. $x^2 + 2x = -2$ **68.** $2x^2 - 3x + 2 = 0$

SECTION 9.5 Graphing Quadratic Equations

DEFINITIONS AND CONCEPTS | **EXAMPLES**

The **vertex** of a parabola is the lowest (or highest) point on the parabola.

A vertical line through the vertex of a parabola that opens upward or downward is called its **axis of symmetry.**



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Equations that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$, are called **quadratic equations in two variables.**

The graph of $y = ax^2 + bx + c$ is a **parabola.** Much can be determined about the graph from the coefficients a, b, and c.

The parabola opens **upward** when $a > 0$ and **downward** when $a < 0$.

The x-coordinate of the **vertex** of the parabola is $x = \frac{-b}{2a}$. To find the y-coordinate of the vertex, substitute $\frac{-b}{2a}$ for x in the equation of the parabola and find y.

To find the ***y*-intercept,** substitute 0 for x in the given equation and solve for y. To find the ***x*-intercepts,** substitute 0 for y in the given equation and solve for x.

The number of distinct x-intercepts of the graph of a quadratic equation $y = ax^2 + bx + c$ is the same as the number of distinct **real-number solutions** of $ax^2 + bx + c = 0$.

Graph: $y = x^2 + 6x + 8$

Upward/downward: The equation is in the form $y = ax^2 + bx + c$, with $a = 1$, $b = 6$, and $c = 8$. Since $a > 0$, the parabola opens upward.

Vertex/axis of symmetry: The x-coordinate of the vertex is

$$\frac{-b}{2a} = \frac{-6}{2(1)} = -3$$

The y-coordinate of the vertex is:

$$y = x^2 + 6x + 8 \quad \text{The equation to graph.}$$
$$y = (-3)^2 + 6(-3) + 8 \quad \text{Substitute } -3 \text{ for } x.$$
$$y = 9 - 18 + 8$$
$$y = -1$$

The vertex of the parabola is $(-3, -1)$.

Intercepts: Since $c = 8$, the y-intercept of the parabola is $(0, 8)$. The point $(-6, 8)$, which is 3 units to the left of the axis of symmetry, must also be on the graph.

To find the x-intercepts of the graph of $y = x^2 + 6x + 8$, we set $y = 0$ and solve for x.

$$0 = x^2 + 6x + 8$$
$$0 = (x + 4)(x + 2)$$
$$x + 4 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = -4 \quad | \quad x = -2$$

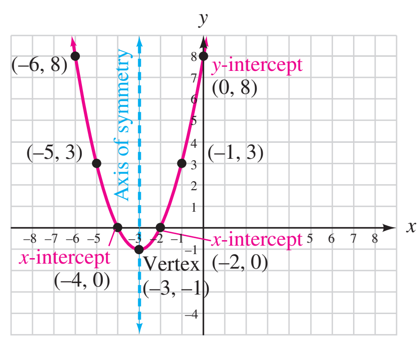
The x-intercepts of the graph are $(-4, 0)$ and $(-2, 0)$.

Plotting points/using symmetry: To locate two more points on the graph, we let $x = -1$ and find the corresponding value of y.

$$y = (-1)^2 + 6(-1) + 8 = 3$$

Thus, the point $(-1, 3)$ lies on the parabola. We use symmetry to determine that $(-5, 3)$ is also on the graph.

Draw a smooth curve through the points: The completed graph of $y = x^2 + 6x + 8$ is shown here.



Since the graph of $y = x^2 + 6x + 8$ (shown above) has two x-intercepts, $(-4, 0)$ and $(-2, 0)$, the equation $x^2 + 6x + 8 = 0$ has two distinct real-number solutions, -4 and -2.

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REVIEW EXERCISES

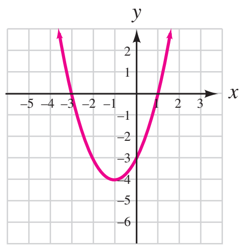
69. Refer to the figure.

a. What are the x-intercepts of the parabola?

b. What is the y-intercept of the parabola?

c. What is the vertex of the parabola?

d. Draw the axis of symmetry of the parabola on the graph.



70. The point $(0, -3)$ lies on the parabola graphed above. Use symmetry to determine the coordinates of another point that lies on the parabola.

Find the vertex of the graph of each quadratic equation and tell in which direction the parabola opens. Do not draw the graph.

71. $y = 2x^2 - 4x + 7$ **72.** $y = -3x^2 + 18x - 11$

Find the x- and y-intercepts of the graph of each quadratic equation.

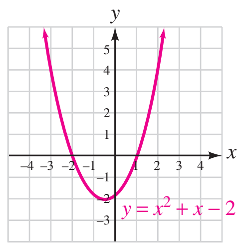
73. $y = x^2 + 6x + 5$ **74.** $y = x^2 + 2x + 3$

Graph each quadratic equation by finding the vertex, the x- and y-intercepts, and the axis of symmetry of its graph.

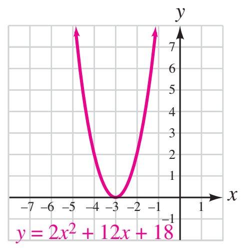
75. $y = x^2 + 2x - 3$ **76.** $y = -2x^2 + 4x - 2$

77. $y = -x^2 - 2x + 5$ **78.** $y = x^2 + 4x - 1$

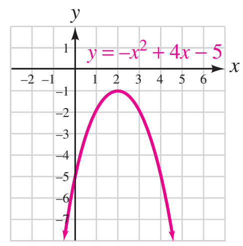
79. The graphs of three quadratic equations in two variables are shown. Fill in the blanks.



$x^2 + x - 2 = 0$ has ____ real-number solution(s). Give the solution(s): ____

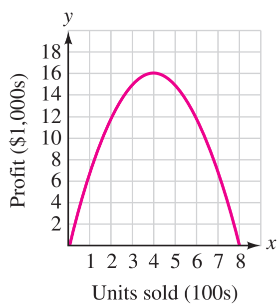


$2x^2 + 12x + 18 = 0$ has ____ repeated real-number solution. Give the solution(s): ____



$-x^2 + 4x - 5 = 0$ has ____ real-number solution(s).

80. **Manufacturing.** What important information can be obtained from the vertex of the parabola in the graph below?



9 CHAPTER TEST

1. Fill in the blanks.

a. A ________ equation can be written in the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.

b. $x^2 + 8x + 16$ is a perfect-_______ trinomial because $x^2 + 8x + 16 = (x + 4)^2$.

c. When we add 25 to $x^2 + 10x$, we say we have _________ the square on $x^2 + 10x$.

d. We read $3 \pm \sqrt{2}$ as "three _____ or ______ the square root of two."

e. The _______ coefficient of $3x^2 + 8x - 9$ is 3 and the ________ term is -9.

2. Write the statement $x = \sqrt{5}$ or $x = -\sqrt{5}$ using double-sign notation.

Solve each equation by the square root method.

3. $x^2 = 17$ **4.** $r^2 - 48 = 0$

5. $(x - 2)^2 = 3$ **6.** $4y^2 - 20 = 5$

7. $t^2 = \frac{1}{49}$ **8.** $x^2 + 16x + 64 = 24$

9. Explain why the equation $m^2 + 49 = 0$ has no real-number solutions.

10. Check to determine whether $4\sqrt{2}$ is a solution of $n^2 - 32 = 0$.

Complete the square and factor the resulting perfect-square trinomial.

11. $x^2 - 14x$

12. $c^2 - 7c$

13. $x^2 + x$

14. $a^2 - \frac{5}{3}a$

15. Complete the square to solve $a^2 + 2a - 4 = 0$. Give the exact solutions and then approximate them to the nearest hundredth.

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16. Complete the square to solve $a^2 + a = 3$.

17. Complete the square to solve $m^2 - 4m + 10 = 0$.

18. Complete the square to solve: $2x^2 = 3x + 2$

Use the quadratic formula to solve each equation.

19. $2x^2 - 5x - 12 = 0$

20. $5x^2 + 11x = -3$

21. $4n^2 - 12n + 1 = 0$

22. $7t^2 = -6t - 4$

23. Solve $3x^2 - 2x - 2 = 0$ using the quadratic formula. Give the exact solutions, and then approximate them to the nearest hundredth.

24. Check to determine whether $1 + \sqrt{5}$ is a solution of $x^2 - 2x - 4 = 0$.

25. **Archery.** The area of a circular archery target is 5,026 cm^2. What is the radius of the target? Round to the nearest centimeter.

26. **St. Louis.** On October 28, 1965, workers "topped out" the final section of the Gateway Arch in St. Louis, Missouri. It is the tallest national monument in the United States at 630 feet. If a worker dropped a tool from that height, how long would it take to reach the ground? Round to the nearest tenth.

AP Photo

27. **New York City.** The rectangular Samsung sign in Times Square is a full color LED screen that has an area of 2,665 ft^2. Its height is 17 feet less than twice its width. Find the width and height of the sign.

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28. **Geometry.** The hypotenuse of a right triangle is 8 feet long. One leg is 4 feet longer than the other. Find the lengths of the legs. Round to the nearest tenth.

Use the most efficient method to solve each equation.

29. $x^2 - 4x = -2$

30. $(3b + 1)^2 = 16$

31. $u^2 - 24 = 0$

32. $6n^2 - 36n = 0$

Write each expression in terms of i.

33. $\sqrt{-100}$

34. $-\sqrt{-18}$

Perform the operations. Write all answers in the form a + bi.

35. $(8 + 3i) + (-7 - 2i)$

36. $(5 + 3i) - (6 - 9i)$

37. $(2 - 4i)(3 + 2i)$

38. $\dfrac{3 - 2i}{3 + 2i}$

Solve each equation. Write all solutions in the form $a + bi$.

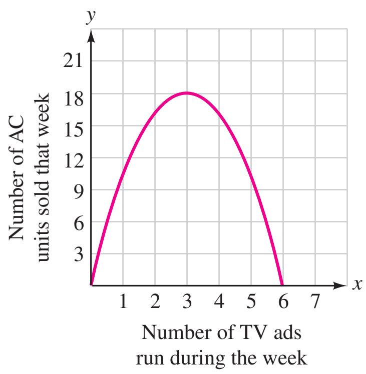
39. $x^2 + 100 = 0$

40. $(a + 3)^2 = -1$

41. $n^2 = -\dfrac{25}{16}$

42. $3x^2 + 2x + 1 = 0$

43. **Advertising.** When a business runs x advertisements per week on television, the number y of air conditioners it sells is given by the graph in the next column. What important information can be obtained from the vertex?



44. Fill in the blanks: The graph of $y = ax^2 + bx + c$ opens downward when a ____ 0 and upward when a ____ 0.

Graph each quadratic equation by finding the vertex, the x- and y-intercepts, and the axis of symmetry of its graph.

45. $y = x^2 + 6x + 5$

46. $y = -x^2 + 6x - 7$

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CUMULATIVE REVIEW Chapters 1–9

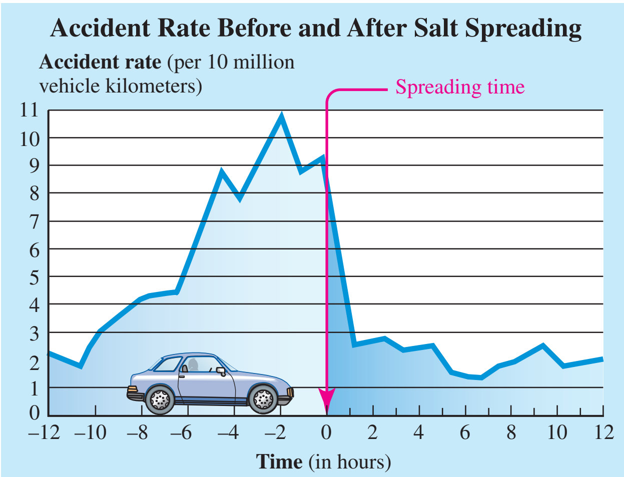
1. Determine whether each statement is true or false. [Section 1.3]

a. Every rational number can be written as a ratio of two integers.

b. The set of real numbers corresponds to all points on the number line.

c. The whole numbers and their opposites form the set of integers.

2. Driving Safety. In cold-weather climates, salt is spread on roads to keep snow and ice from bonding to the pavement. This allows snowplows to remove built-up snow quickly. According to the graph, when is the accident rate the highest? [Section 1.4]



Based on data from a study done in Europe by the Salt Institute

3. Evaluate: $-4 + 2[-7 - 3(-9)]$ [Section 1.7]

4. Evaluate: $\left|\frac{4}{5} \cdot 10 - 12\right|$ [Section 1.7]

5. Evaluate $(x - a)^2 + (y - b)^2$ for $x = -2, y = 1, a = 5$, and $b = -3$. [Section 1.8]

6. Simplify: $3p - 6(p - 9) + p$ [Section 1.9]

7. Solve $\frac{5}{6}k = 10$ and check the result. [Section 2.2]

8. Solve $-(3a + 1) + a = 2$ and check the result. [Section 2.2]

9. Loose Change. The Coinstar machines that are in many grocery stores count unsorted coins and print out a voucher that can be exchanged for cash at the checkout stand. However, to use this service, a processing fee is charged. If a boy turned in a jar of coins worth \$60 and received a voucher for \$54.12, what was the processing fee (expressed as a percent) charged by Coinstar? [Section 2.3]

10. Solve $T = 2r + 2t$ for r. [Section 2.4]

11. Selling a Home. At what price should a home be listed if the owner wants to make \$330,000 on its sale after paying a 4% real estate commission? [Section 2.5]

12. Business Loans. Last year, a women's professional organization made two small-business loans totaling \$28,000 to young women beginning their own businesses. The money was lent at 7% and 10% simple interest rates. If the annual income the organization received from these loans was \$2,560, what was each loan amount? [Section 2.6]

13. Solve $5x + 7 < 2x + 1$ and graph the solution set. Then use interval notation to describe the solution. [Section 2.7]

14. Check to determine whether $(-5, -3)$ is a solution of $2x - 3y = -1$. [Section 3.1]

Graph each equation or inequality.

15. $y = -x + 2$ [Section 3.2]

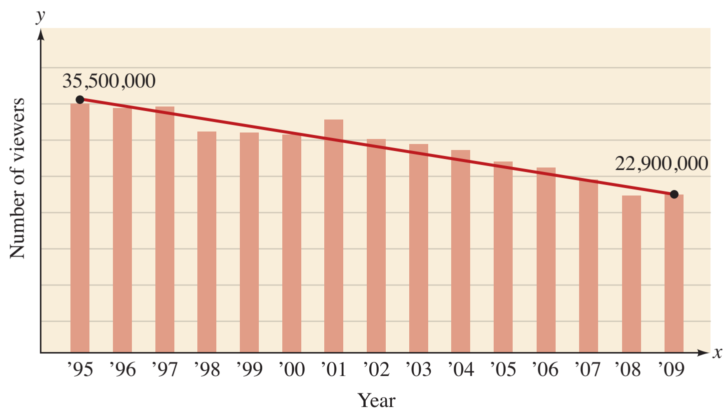
16. $2y - 2x = 6$ [Section 3.3]

17. $y = -3$ [Section 3.3]

18. $y < 3x$ [Section 3.7]

19. Find the slope of the line passing through $(-2, -2)$ and $(-12, -8)$. [Section 3.4]

20. TV News. The line graph in red below approximates the evening news viewership on all networks for the years 1995–2009. Find the rate of decrease over this period of time. [Section 3.4]



Source: The State of the News Media, 2010

21. What is the slope of the line defined by $4x + 5y = 6$? [Section 3.5]

22. Write the equation of the line whose graph has slope -2 and y-intercept $(0, 1)$. [Section 3.5]

23. Are the graphs of $y = 4x + 9$ and $x + 4y = -10$ parallel, perpendicular, or neither? [Section 3.5]

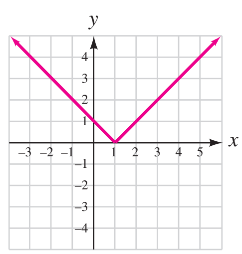
24. Write the equation of the line whose graph has slope $\frac{1}{4}$ and passes through the point $(8, 1)$. Write the equation in slope–intercept form. [Section 3.6]

25. Graph the line passing through $(-2, -1)$ and having slope $\frac{4}{3}$. [Section 3.6]

26. If $f(x) = 3x^2 + 3x - 8$, find $f(-1)$. [Section 3.8]

27. Find the domain and range of the relation: $\{(1, 8), (4, -3), (-4, 2), (5, 8)\}$ [Section 3.8]

28. Is this the graph of a function? [Section 3.8]



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29. Solve using the graphing method. [Section 4.1]

$$\begin{cases} x + y = 1 \\ y = x + 5 \end{cases}$$

30. Solve using the substitution method.

$$\begin{cases} y = 2x + 5 \\ x + 2y = -5 \end{cases}$$ [Section 4.2]

31. Solve using the elimination (addition) method.

$$\begin{cases} \frac{3}{5}s + \frac{4}{5}t = 1 \\ -\frac{1}{4}s + \frac{3}{8}t = 1 \end{cases}$$ [Section 4.3]

32. Aviation. With the wind, a plane can fly 3,000 miles in 5 hours. Against the wind, the trip takes 6 hours. Find the airspeed of the plane (the speed in still air). Use two variables to solve this problem. [Section 4.4]

33. Mixing Candy. How many pounds of each candy must be mixed to obtain 48 pounds of candy that would be worth \$4.50 per pound? Use two variables to solve this problem. [Section 4.4]

34. Solve the system of linear inequalities.

$$\begin{cases} 3x + 4y > -7 \\ 2x - 3y \geq 1 \end{cases}$$ [Section 4.5]

Simplify each expression. Write each answer without using parentheses or negative exponents.

35. $y^3(y^2y^4)$ [Section 5.1]

36. $\left(\frac{b^2}{3a}\right)^3$ [Section 5.1]

37. $\frac{10a^4a^{-2}}{5a^2a^0}$ [Section 5.2]

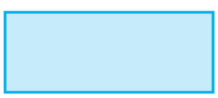
38. $\left(\frac{21x^{-2}y^2z^{-2}}{7x^3y^{-1}}\right)^{-2}$ [Section 5.2]

39. Five-Card Poker. The odds against being dealt the hand shown are about 2.6×10^6 to 1. Express 2.6×10^6 using standard notation. [Section 5.3]

40. Write 0.00073 in scientific notation. [Section 5.3]

41. Graph: $y = x^3 - 2$ [Section 5.4]

42. Write a polynomial that represents the perimeter of the rectangle. [Section 5.5]



Perform the operations.

43. $4(4x^3 + 2x^2 - 3x - 8) - 5(2x^3 - 3x + 8)$ [Section 5.5]

44. $(-2a^3)(3a^2)$ [Section 5.6]

45. $(2b - 1)(3b + 4)$ [Section 5.6]

46. $(3x + y)(2x^2 - 3xy + y^2)$ [Section 5.6]

47. $(2x + 5y)^2$ [Section 5.7]

48. $(9m^2 - 1)(9m^2 + 1)$ [Section 5.7]

49. $\frac{12a^3b - 9a^2b^2 + 3ab}{6a^2b}$ [Section 5.8]

50. $x - 3\overline{)2x^2 - 3 - 5x}$ [Section 5.8]

Factor each expression completely.

51. $6a^2 - 12a^3b + 36ab$ [Section 6.1]

52. $2x + 2y + ax + ay$ [Section 6.1]

53. $x^2 - 6x - 16$ [Section 6.2]

54. $30y^5 + 63y^4 - 30y^3$ [Section 6.3]

55. $t^4 - 16$ [Section 6.4]

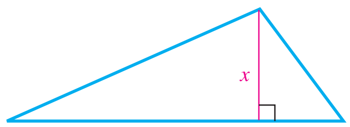
56. $b^3 + 125$ [Section 6.5]

Solve each equation by factoring.

57. $3x^2 + 8x = 0$ [Section 6.7]

58. $15x^2 - 2 = 7x$ [Section 6.7]

59. Geometry. The triangle shown has an area of 22.5 square inches. Find its height. [Section 6.8]



60. For what value is $\frac{x}{x + 8}$ undefined? [Section 7.1]

Simplify each expression.

61. $\frac{3x^2 - 27}{x^2 + 3x - 18}$ [Section 7.1]

62. $\frac{a - 15}{15 - a}$ [Section 7.1]

Perform the operations and simplify when possible.

63. $\frac{x^2 - x - 6}{2x^2 + 9x + 10} \div \frac{x^2 - 25}{2x^2 + 15x + 25}$ [Section 7.2]

64. $\frac{1}{s^2 - 4s - 5} + \frac{s}{s^2 - 4s - 5}$ [Section 7.3]

65. $\frac{x + 5}{xy} - \frac{x - 1}{x^2y}$ [Section 7.4]

66. $\frac{x}{x - 2} + \frac{3x}{x^2 - 4}$ [Section 7.4]

Simplify each complex fraction.

67. $\frac{\frac{9m - 27}{m^6}}{\frac{2m - 6}{m^8}}$ [Section 7.5]

68. $\frac{\frac{5}{y} + \frac{4}{y + 1}}{\frac{4}{y} - \frac{5}{y + 1}}$ [Section 7.5]

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Solve each equation.

69. $\frac{2p}{3} - \frac{1}{p} = \frac{2p - 1}{3}$ [Section 7.6]

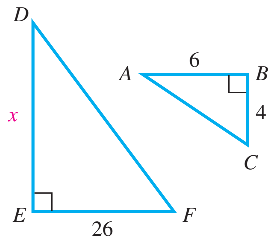
70. $\frac{7}{q^2 - q - 2} + \frac{1}{q + 1} = \frac{3}{q - 2}$ [Section 7.6]

71. Solve the formula $\frac{1}{a} + \frac{1}{b} = 1$ for a. [Section 7.6]

72. **Roofing.** A homeowner estimates that it will take him 7 days to roof his house. A professional roofer estimates that he can roof the house in 4 days. How long will it take if the homeowner helps the roofer? [Section 7.7]

73. **Losing Weight.** If a person cuts his or her daily calorie intake by 100, it will take 350 days for that person to lose 10 pounds. How long will it take for the person to lose 25 pounds? [Section 7.8]

74. ΔABC and ΔDEF are similar triangles. Find x. [Section 7.8]



75. Suppose w varies directly as x. If $w = 1.2$ when $x = 4$, find w when $x = 30$. [Section 7.9]

76. **Gears.** The speed of a gear varies inversely with the number of teeth. If a gear with 10 teeth makes 3 revolutions per second, how many revolutions per second will a gear with 25 teeth make? [Section 7.9]

Simplify each radical expression. All variables represent positive numbers.

77. $\sqrt{100x^2}$ [Section 8.1]

78. $-\sqrt{18b^3}$ [Section 8.2]

Perform the indicated operation.

79. $3\sqrt{24} + \sqrt{54}$ [Section 8.3]

80. $(\sqrt{2} + 1)(\sqrt{2} - 3)$ [Section 8.4]

Rationalize the denominator.

81. $\frac{8}{\sqrt{10}}$ [Section 8.4]

82. $\frac{\sqrt{2}}{3 - \sqrt{a}}$ [Section 8.4]

Solve each equation.

83. $\sqrt{6x + 1} + 2 = 7$ [Section 8.5]

84. $\sqrt{3t + 7} = t + 3$ [Section 8.5]

Simplify each radical expression. All variables represent positive numbers.

85. $\sqrt[3]{\frac{27m^3}{8n^6}}$ [Section 8.6]

86. $\sqrt[4]{16}$ [Section 8.6]

Evaluate each expression.

87. $25^{3/2}$ [Section 8.6]

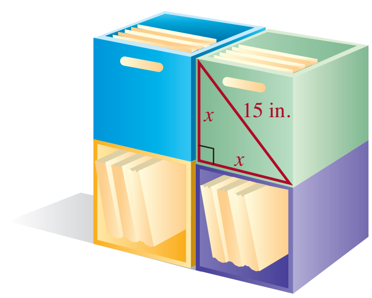
88. $(-8)^{-4/3}$ [Section 8.6]

Solve each equation.

89. $t^2 = 75$ [Section 9.1]

90. $(6y + 5)^2 - 72 = 0$ [Section 9.1]

91. **Storage Cubes.** The diagonal distance across the face of each of the stacking cubes is 15 inches. What is the height of the entire storage arrangement? Round to the nearest tenth of an inch. [Section 9.1]



92. Solve $x^2 + 8x + 12 = 0$ by completing the square. [Section 9.2]

93. Solve $4x^2 - x - 2 = 0$ using the quadratic formula. Give the exact solutions, and then approximate each to the nearest hundredth. [Section 9.3]

94. **Quilts.** According to the *Guinness Book of World Records 1998,* the world's largest quilt was made by the Seniors' Association of Saskatchewan, Canada, in 1994. If the length of the rectangular quilt is 11 feet less than twice its width and it has an area of 12,865 ft^2, find its width and length. [Section 9.3]

Write each expression in terms of i.

95. $\sqrt{-49}$ [Section 9.4]

96. $\sqrt{-54}$ [Section 9.4]

Perform the operations. Express each answer in the form a + bi.

97. $(2 + 3i) - (1 - 2i)$ [Section 9.4]

98. $(7 - 4i) + (9 + 2i)$ [Section 9.4]

99. $(3 - 2i)(4 - 3i)$ [Section 9.4]

100. $\frac{3 - i}{2 + i}$ [Section 9.4]

Solve each equation. Express the solutions in the form a + bi.

101. $x^2 + 16 = 0$ [Section 9.4]

102. $x^2 - 4x = -5$ [Section 9.4]

103. Graph the quadratic equation $y = 2x^2 + 8x + 6$. Find the vertex, the x- and y-intercepts, and the axis of symmetry of the graph. [Section 9.5]

104. **Power Output.** The graph shows the power output (in horsepower, hp) of a certain engine for various engine speeds (in revolutions per minute, rpm). For what engine speed does the power output reach a maximum? [Section 9.5]