Complex Numbers and Powers of *i*

The Number *i* - i is the unique number for which $i = \sqrt{-1}$ and $i^2 = -1$.

Imaginary Number – any number that can be written in the form $a + bi$, where a and b are real numbers and $b \neq 0$.

Complex Number – any number that can be written in the form $a + bi$, where a and b are real numbers. (Note: a and b both **can** be 0.) *The union of the set of all imaginary numbers and the set of all real numbers is the set of complex numbers.*

Addition / Subtraction - Combine like terms (i.e. the real parts with real parts and the imaginary parts with imaginary parts).

Example - $(2 - 3i) - (4 - 6i) = 2 - 3i - 4 + 6i$

$= -2 + 3i$

Multiplication - When multiplying square roots of negative real numbers, begin by expressing them in terms of i.

Example - $\sqrt{-4} \cdot \sqrt{-8} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{-1} \cdot \sqrt{8}$

$= i \cdot 2 \cdot i \cdot 2\sqrt{2}$

$= i^2 \cdot 4\sqrt{2}$

$= (-1) \cdot 4\sqrt{2}$

$= -4\sqrt{2}$

Note: The answer is not $+4\sqrt{2}$, which could be calculated erroneously if the radicands were simply multiplied as $\sqrt{-4} \cdot \sqrt{-8} \neq \sqrt{(-4)(-8)} \neq \sqrt{32}$

Multiplication (Cont'd) – When multiplying two complex numbers, begin by F O I L ing them together and then simplify.

Example - $(2+3i)\cdot(8-7i) = 16-14i+24i-21i^2$

$= 16+10i-21i^2$

$= 16+10i-21\,(-1)$

$= 16+10i+21$

$= 37+10i$

Division – When dividing by a complex number, multiply the top and bottom by the complex conjugate of the denominator. Then F O I L the top and the bottom and simplify. The answer should be written in standard form $(a+bi.)$

Example - $\frac{2+3i}{1-5i} = \frac{(2+3i)}{(1-5i)}\cdot\frac{(1+5i)}{(1+5i)}$ (Multiply by complex conjugate)

$= \frac{2+10i+3i+15\,i^2}{1+5i-5i-25\,i^2} = \frac{2+13i+15(-1)}{1-25(-1)}$

$= \frac{2+13i-15}{1+25} = \frac{-13+13i}{26}$

$= \frac{-1+i}{2} = \frac{-1}{2} + \frac{1}{2}\,i$

Example - $\frac{14}{i} = \frac{14}{i}\cdot\frac{-i}{-i}$ (Multiply by complex conjugate)

$= \frac{-14\,i}{-i^2} = \frac{-14i}{-(-1)}$

$= \frac{-14i}{1} = -14\,i$

Powers of $\boldsymbol{i}$ – Given a number, i^n, the number can be simplified by using the following chart.

$\boldsymbol{i^n}$	**Is Equivalent to…**	**Because…**
i^0	1	$a\ number\ raised\ to\ the\ 0\ power\ is\ 1$
i^1	i	$a\ number\ raised\ to\ the\ 1\ power\ is\ that\ same\ number$
i^2	(-1)	$i^2 = -1$ *(definition of i)*
i^3	$-i$	$i^3 = i^2 \cdot i = (-1) \cdot i = -i$
i^4	1	$i^4 = i^2 \cdot i^2 = (-1) \cdot (-1) = 1$
i^5	i	$i^5 = i^4 \cdot i = (1) \cdot i = i$

Because the powers of i will cycle through $1, i, -1, and - i$, this repeating pattern of four terms can be used to simplify i^n.

Example - Simplify i^{25}

Step 1 - Divide 25 *(the power)* by 4.

$\frac{25}{4}$ = quotient of 6 with a remainder of 1

Step 2 - Note the quotient (i.e. 6) and the remainder (i.e. 1).

Step 3 - Rewrite the problem.

$$i^{25} = (i^4)^{quotient} \cdot i^{remainder} = (i^4)^6 \cdot i^1$$

Step 4 - Simplify by recalling that $i^4 = 1$

$$(i^4)^6 \cdot i^1 = (1)^6 \cdot i^1 = 1 \cdot i = i$$

Note: Because the powers of i cycle through $1, i, -1, and\ i$, these types of problems can always be simplified by noting what the <u>remainder</u> is in step 2 above. In fact, the problem can be re-written as…

$i^n = i^{remainder}$ (Divide n by 4 and determine the remainder).

The remainder will always be either 0, 1, 2, or 3.

Example - Simplify i^{59}

$i^{59} = i^3$ *(because* $\frac{59}{4}$ *has a remainder of 3.)*

So, $i^{59} = i^3 = -i$

Imaginary and Complex Numbers Practice

Simplify:

1) $(4 + 2i) + (-3 - 5i)$
2) $(-3 + 4i) - (5 + 2i)$
3) $(-8 - 7i) - (5 - 4i)$
4) $(3 - 2i)(5 + 4i)$
5) $(3 - 4i)^2$
6) $(3 - 2i)(5 + 4i) - (3 - 4i)^2$
7) Write $\frac{3+7i}{5-3i}$ in standard form
8) Simplify i^{925}
9) Simplify i^{460}
10) Write $\frac{1-4i}{5+2i}$ in standard form
11) $\sqrt{-16}$
12) $\sqrt{-8}$
13) $\sqrt{-6}\ \sqrt{-6}$
14) $4 + \sqrt{-25}$
15) $\frac{6-\sqrt{-8}}{-2}$

Answers:

(1) $1 - 3i$	(2) $-8 + 2i$	(3) $-13 - 3i$	(4) $23 + 2i$
(5) $-7 - 24i$	(6) $30 + 26i$	(7) $\frac{-3}{17}+\frac{22}{17}i$	(8) i
(9) 1	(10) $\frac{-2}{29}-\frac{22}{29}i$	(11) $4i$	(12) $2\sqrt{2}\,i$
(13) -6	(14) $4 + 5i$	(15) $-3 + \sqrt{2}\,i$	