Generating Equivalent Algebraic Expressions

MODULE 11

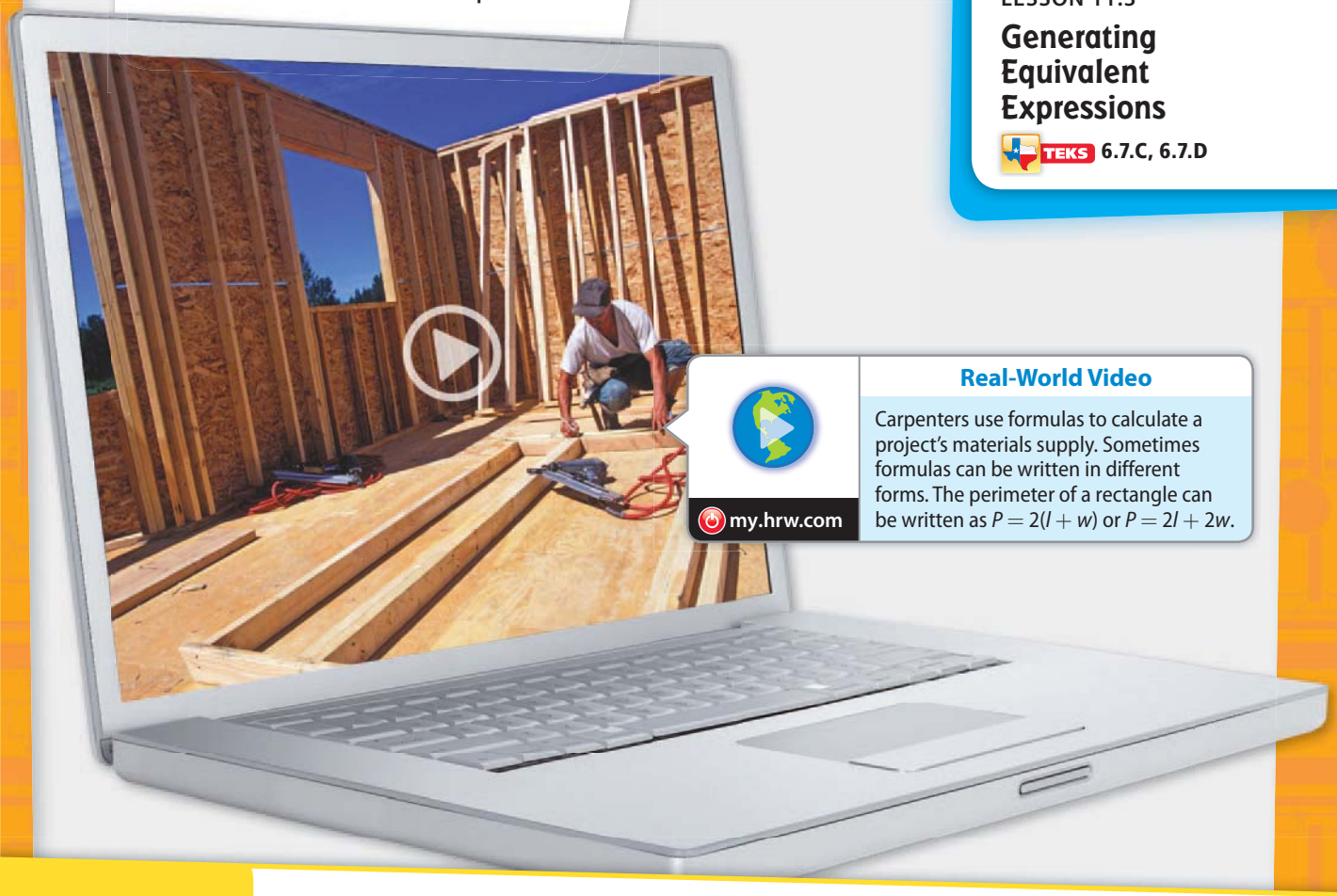
LESSON 11.1
Modeling Equivalent Expressions
TEKS 6.7.C

LESSON 11.2
Evaluating Expressions
TEKS 6.7.A

LESSON 11.3
Generating Equivalent Expressions
TEKS 6.7.C, 6.7.D

ESSENTIAL QUESTION

How can you generate equivalent algebraic expressions and use them to solve real-world problems?



my.hrw.com

Real-World Video

Carpenters use formulas to calculate a project's materials supply. Sometimes formulas can be written in different forms. The perimeter of a rectangle can be written as $P = 2(l + w)$ or $P = 2l + 2w$.

GO DIGITAL
my.hrw.com

my.hrw.com
Go digital with your write-in student edition, accessible on any device.

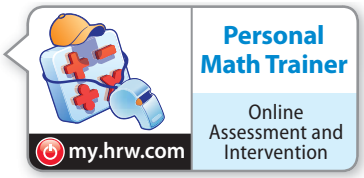
Math On the Spot
Scan with your smart phone to jump directly to the online edition, video tutor, and more.

Animated Math
Interactively explore key concepts to see how math works.

Personal Math Trainer
Get immediate feedback and help as you work through practice sets.

Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.



Use of Parentheses

EXAMPLE $(6 + 4) \times (3 + 8 + 1) = 10 \times 12$ *Do the operations inside parentheses first.*

$= 120$ *Multiply.*

Evaluate.

1. $11 + (20 - 13)$ _____

2. $(10 - 7) - (14 - 12)$ _____

3. $(4 + 17) - (16 - 9)$ _____

4. $(23 - 15) - (18 - 13)$ _____

5. $8 \times (4 + 5 + 7)$ _____

6. $(2 + 3) \times (11 - 5)$ _____

Words for Operations

EXAMPLE Write a numerical expression for the quotient of 20 and 5. *Think: Quotient means to divide.*

$20 \div 5$ *Write 20 divided by 5.*

Write a numerical expression for the word expression.

7. the difference between 42 and 19 _____

8. the product of 7 and 12 _____

9. 30 more than 20 _____

10. 100 decreased by 77 _____

Evaluate Expressions

EXAMPLE Evaluate $2(5) - 3^2$.

$2(5) - 3^2 = 2(5) - 9$ *Evaluate exponents.*

$= 10 - 9$ *Multiply.*

$= 1$ *Subtract.*

Evaluate the expression.

11. $3(8) - 15$ _____

12. $4(12) + 11$ _____

13. $3(7) - 4(2)$ _____

14. $4(2 + 3) - 12$ _____

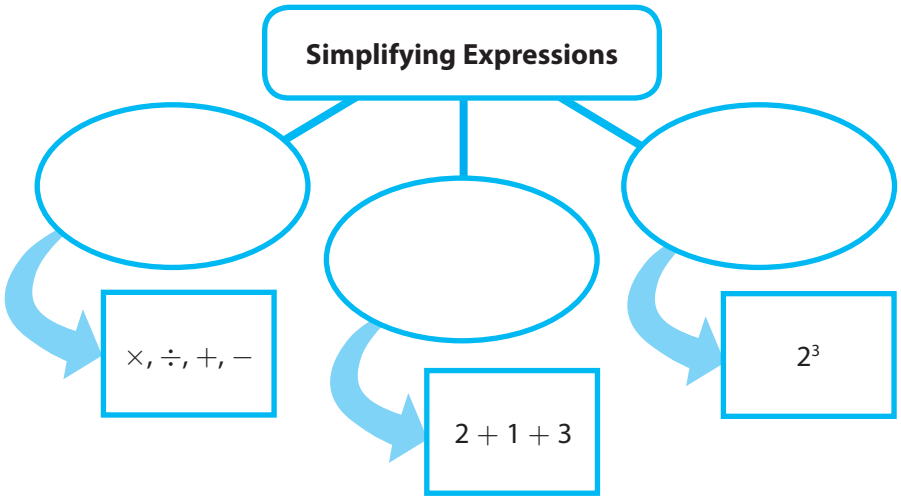
15. $9(14 - 5) - 42$ _____

16. $7(8) - 5(8)$ _____

Reading Start-Up

Visualize Vocabulary

Use the review words to complete the graphic. You may put more than one word in each oval.



Understand Vocabulary

Complete the sentences using the preview words.

1. An expression that contains at least one variable is an _____ .
2. A part of an expression that is added or subtracted is a _____ .
3. A _____ is a specific number whose value does not change.

Vocabulary

Review Words

- base *(base)*
- exponent *(exponente)*
- numerical expression *(expresión numérica)*
- operations *(operaciones)*
- order of operations *(orden de las operaciones)*

Preview Words

- algebraic expression *(expresión algebraica)*
- coefficient *(coeficiente)*
- constant *(constante)*
- equivalent expression *(expresión equivalente)*
- evaluating *(evaluar)*
- like terms *(términos semejantes)*
- term *(término, en una expresión)*
- variable *(variable)*

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.

MODULE 11

Unpacking the TEKS

Understanding the TEKS and the vocabulary terms in the TEKS will help you know exactly what you are expected to learn in this module.

TEKS 6.7.C

Determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations.

Key Vocabulary

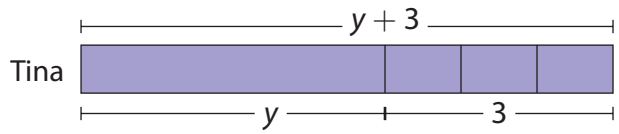
equivalent expressions
(expresión equivalente)
Expressions that have the same value for all values of the variables.

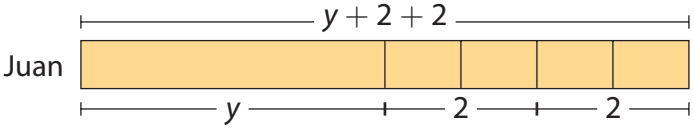
What It Means to You

You will use models to compare expressions.

UNPACKING EXAMPLE 6.7.C

On a math quiz, Tina scored 3 points more than Yolanda. Juan scored 2 points more than Yolanda and earned 2 points as extra credit. Draw models for Tina's and Juan's scores. Use your models to decide whether they made the same score.





Tina and Juan did not make the same score because the models do not show equivalent expressions.

TEKS 6.7.D

Generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

What It Means to You

You will use the properties of operations to find an equivalent expression.

UNPACKING EXAMPLE 6.7.D

William earns $13 an hour working at a movie theater. He worked h hours in concessions and three times as many hours at the ticket counter. Write and simplify an expression for the amount of money William earned.

$13 · hours at concessions + $13 · hours at ticket counter

$13h + 13(3h)$

$13h + 39h$ Multiply $13 · 3h$.

$h(13 + 39)$ Distributive Property

Visit my.hrw.com to see all the TEKS unpacked.

my.hrw.com

LESSON 11.1

Modeling Equivalent Expressions

TEKS **Expressions, equations, and relationships—6.7.C** Determine if two expressions are equivalent using concrete models, pictorial models, and algebraic representations.

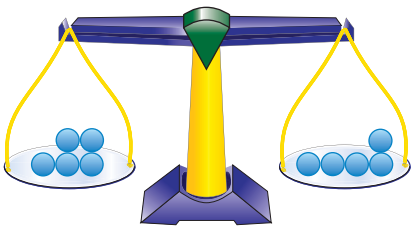
ESSENTIAL QUESTION

How can you write algebraic expressions and use models to decide if expressions are equivalent?

EXPLORE ACTIVITY TEKS 6.7.C

Modeling Equivalent Expressions

Equivalent expressions are expressions that have the same value.



The scale shown to the right is balanced.

A Write an expression to represent the circles on the left side of the balance. _____

B The value of the expression on the left side is _____.

C Write an expression to represent the circles on the right side of the balance. _____

D The value of the expression on the right side is _____.

E Since the expressions have the same value, the expressions are _____.

F What will happen if you remove a circle from the right side of the balance?

_____

G If you add a circle to the left side of the balance, what can you do to the right side to keep the scale in balance?

_____

Reflect

1. **What If?** Suppose there were $2 + 5$ circles on the right side of the balance and 3 on the left side of the balance. What can you do to balance the scale? Explain how the scale models equivalent expressions.

_____

_____

Math On the Spot
my.hrw.com

Writing Algebraic Expressions

An **algebraic expression** is an expression that contains one or more variables and may also contain operation symbols, such as $+$ or $-$.

A **variable** is a letter or symbol used to represent an unknown or unspecified number. The value of a variable may change.

A **constant** is a specific number whose value does not change.

constant variable

Algebraic Expressions	$150 + y$	$w + n$	x
Not Algebraic Expressions	15	$12 - 7$	$\frac{9}{16}$

In algebraic expressions, multiplication and division are usually written without the symbols $\times$ and $\div$.

- Write $3 \times n$ as $3n$, $3 \cdot n$, or $n \cdot 3$.
- Write $3 \div n$ as $\frac{3}{n}$.

There are several different ways to describe expressions with words.

Operation	Addition	Subtraction	Multiplication	Division
Words	• added to • plus • sum • more than	• subtracted from • minus • difference • less than • take away • taken from	• times • multiplied by • product • groups of	• divided by • divided into • quotient

EXAMPLE 1

TEKS 6.7.C

A **Write each phrase as an algebraic expression.**

The sum of 7 and x — *The operation is addition.*

The algebraic expression is $7 + x$.

The quotient of z and 3 — *The operation is division.*

The algebraic expression is $\frac{z}{3}$.

B **Write a phrase for each expression.**

$11x$ — *The operation is multiplication.*

The product of 11 and x

$8 - y$ — *The operation is subtraction.*

y less than 8

YOUR TURN

Write each phrase as an algebraic expression.

2. *n* times 7 _____ **3.** 4 minus *y* _____ **4.** 13 added to *x* _____

Write a phrase for each expression.

5. $\frac{x}{12}$ _____ **6.** $10y$ _____

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Modeling Algebraic Expressions

Algebraic expressions can also be represented with models.

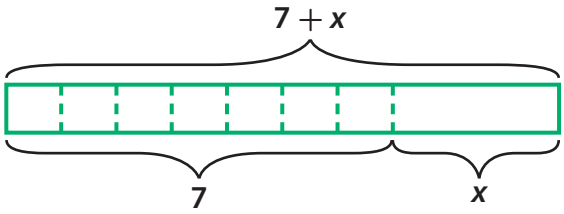
Math On the Spot
my.hrw.com

EXAMPLE 2

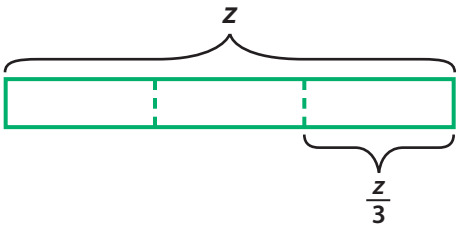
TEKS 6.7.C

Use a bar model to represent each expression.

A $7 + x$ *Combine 7 and x.*



B $\frac{z}{3}$ *Divide z into 3 equal parts.*



Math Talk
Mathematical Processes

What two phrases can you use to describe the expression $\frac{4}{x}$? What is different about the two phrases?

YOUR TURN

Draw a bar model to represent each expression.

7. $t - 2$

8. $4y$

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Math On the Spot
my.hrw.com

Comparing Expressions Using Models

Algebraic expressions are *equivalent* if they are equal for all values of the variable. For example, $x + 2$ and $x + 1 + 1$ are equivalent.

EXAMPLE 3

Real World

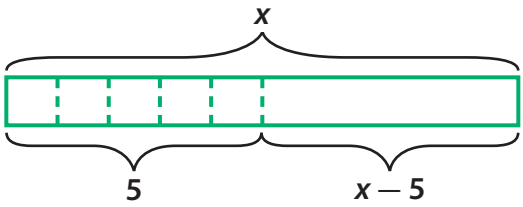
TEKS 6.7.C

Katriana and Andrew started the day with the same amount of money. Katriana spent 5 dollars on lunch. Andrew spent 3 dollars on lunch and 2 dollars on an afterschool snack.

Do Katriana and Andrew have the same amount of money left?

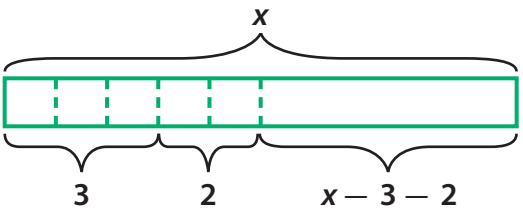
The variable represents the amount of money both Katriana and Andrew have at the beginning of the day.

STEP 1 Write an algebraic expression to represent the money Katriana has left. Represent the expression with a model.



$x - 5$

STEP 2 Write an algebraic expression to represent the money Andrew has left. Represent the expression with a model.



$x - 3 - 2$

STEP 3 Compare the models.

The models are equivalent, so the expressions are equivalent.

Andrew and Katriana have the same amount of money left.

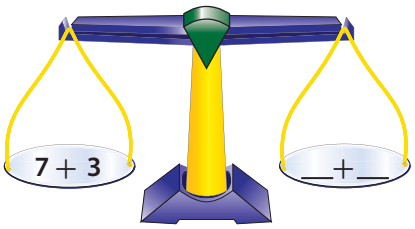
YOUR TURN

9. On a math quiz, Tina scored 3 points more than Julia. Juan scored 2 points more than Julia and earned 2 points in extra credit. Write an expression and draw a bar model to represent Tina's score and Juan's score. Did Tina and Juan make the same grade on the quiz? Explain.

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Guided Practice

1. Write an expression in the right side of the scale that will keep the scale balanced. (Explore Activity)



Write each phrase as an algebraic expression. (Example 1)

2. 3 less than y _____

3. The product of 2 and p _____

Write a phrase for each algebraic expression. (Example 1)

4. $y + 12$ _____

5. $\frac{p}{10}$ _____

6. Draw a bar model to represent the expression $m \div 4$. (Example 2)

At 6 p.m., the temperature in Phoenix, AZ, *t*, is the same as the temperature in Tucson, AZ. By 9 p.m., the temperature in Phoenix has dropped 2 degrees and in Tucson it has dropped 4 degrees. By 11 p.m., the temperature in Phoenix has dropped another 3 degrees. (Example 3)

7. Represent the temperature in each city with an algebraic expression and a bar model.

t

_____

t

_____

8. Are the expressions that represent the temperatures in the two cities equivalent? Justify your answer.

_____

_____

ESSENTIAL QUESTION CHECK-IN

9. How can you use expressions and models to determine if expressions are equivalent?

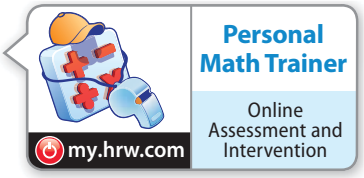
_____

_____

Name _____ Class _____ Date _____

11.1 Independent Practice

TEKS 6.7.C



10. Write an algebraic expression with the constant 7 and the variable *y*.

_____

11. Write an algebraic expression with two variables and one constant.

_____

12. What are the variables in the expression $x + 8 - y$?

_____

13. Identify the parts of the algebraic expression $x + 15$.

Constant(s) _____

Variable(s) _____

Write each phrase as an algebraic expression.

14. *n* divided by 8 _____

15. *p* multiplied by 4 _____

16. *b* plus 14 _____

17. 90 times *x* _____

18. *a* take away 16 _____

19. *k* less than 24 _____

20. 3 groups of *w* _____

21. the sum of 1 and *q* _____

22. the quotient of 13 and *z* _____

23. *c* added to 45 _____

Write a phrase in words for each algebraic expression.

24. $m + 83$ _____

25. $42s$ _____

26. $\frac{9}{d}$ _____

27. $t - 29$ _____

28. $2 + g$ _____

29. $11x$ _____

30. $\frac{h}{12}$ _____

31. $5 - k$ _____

Sarah and Noah work at Read On Bookstore and get paid the same hourly wage. The table shows their work schedule for last week.

Read On Bookstore Work Schedule (hours)			
	Monday	**Tuesday**	**Wednesday**
Sarah	5	3	
Noah			8

32. Write an expression that represents Sarah's total pay last week. Represent her hourly wage with *w*.

_____

33. Write an expression that represents Noah's total pay last week. Represent his hourly wage with *w*.

_____

34. Are the expressions equivalent? Did Sarah and Noah earn the same amount last week? Use models to justify your answer.

_____

_____

_____

_____

_____

35. **Critique Reasoning** Lisa concluded that $3 \cdot 2$ and 3^2 are equivalent expressions. Is Lisa correct? Explain.

36. **Multiple Representations** How could you represent the expressions $x - 5$ and $x - 3 - 3$ on a scale like the one you used in the Explore Activity? Would the scale balance?

37. **Multistep** Will, Hector, and Lydia volunteered at the animal shelter in March and April. The table shows the number of hours Will and Hector volunteered in March. Let x represent the number of hours Lydia volunteered in March.

March Volunteering	
Will	3 hours
Hector	5 hours

a. Will's volunteer hours in April were equal to his March volunteer hours plus Lydia's March volunteer hours. Write an expression to represent Will's volunteer hours in April.

b. Hector's volunteer hours in April were equal to 2 hours less than his March volunteer hours plus Lydia's March volunteer hours. Write an expression to represent Hector's volunteer hours in April.

c. Did Will and Hector volunteer the same number of hours in April? Explain.

38. The town of Rayburn received 6 more inches of snow than the town of Greenville. Let g represent the amount of snow in Greenville. Write an algebraic expression to represent the amount of snow in Rayburn.

39. Abby baked 48 cookies and divided them evenly into bags. Let b represent the number of bags. Write an algebraic expression to represent the number of cookies in each bag.

40. Eli is driving at a speed of 55 miles per hour. Let h represent the number of hours that Eli drives at this speed. Write an algebraic expression to represent the number of miles that Eli travels during this time.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

41. **Represent Real-World Problems** If the number of shoes in a closet is s, then how many pairs of shoes are in the closet? Explain.

42. **Communicate Mathematical Ideas** Is $12x$ an algebraic expression? Explain why or why not.

43. **Problem Solving** Write an expression that has three terms, two different variables, and one constant.

44. **Represent Real-World Problems** Describe a situation that can be modeled by the expression $x - 8$.

45. **Critique Reasoning** Ricardo says that the expression $y + 4$ is equivalent to the expression $1y + 4$. Is he correct? Explain.

LESSON

11.2 Evaluating Expressions

TEKS Expressions, equations, and relationships—6.7.A
Generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization.

ESSENTIAL QUESTION

How can you use the order of operations to evaluate algebraic expressions?

Evaluating Expressions

Recall that an algebraic expression contains one or more variables. You can substitute a number for that variable and then find the value of the expression. This is called **evaluating** the expression.

Math On the Spot
my.hrw.com

EXAMPLE 1

TEKS 6.7.A

Evaluate each expression for the given value of the variable.

A $x - 9; x = 15$

$15 - 9$ Substitute 15 for x.

6 Subtract.

When $x = 15$, $x - 9 = 6$.

B $\frac{16}{n}; n = 8$

$\frac{16}{8}$ Substitute 8 for n.

2 Divide.

When $n = 8$, $\frac{16}{n} = 2$.

C $0.5y; y = 1.4$

$0.5(1.4)$ Substitute 1.4 for y.

0.7 Multiply.

When $y = 1.4$, $0.5y = 0.7$.

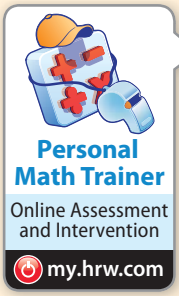
D $6k; k = \frac{1}{3}$

HINT: Think of 6 as $\frac{6}{1}$.

$6\left(\frac{1}{3}\right)$ Substitute $\frac{1}{3}$ for k.

2 Multiply.

When $k = \frac{1}{3}$, $6k = 2$.



YOUR TURN

Evaluate each expression for the given value of the variable.

1. $4x; x = 8$ _____ **2.** $6.5 - n; n = 1.8$ _____ **3.** $\frac{m}{6}; m = 18$ _____

Using the Order of Operations

Expressions may have more than one operation or more than one variable. To evaluate these expressions, substitute the given value for each variable and then use the order of operations.

EXAMPLE 2

TEKS 6.7.A

Evaluate each expression for the given value of the variable.

A $4(x - 4); x = 7$

$4(7 - 4)$	Substitute 7 for x.
$4(3)$	Subtract inside the parentheses.
12	Multiply.

When $x = 7$, $4(x - 4) = 12$.

B $4x - 4; x = 7$

$4(7) - 4$	Substitute 7 for x.
$28 - 4$	Multiply.
24	Subtract.

When $x = 7$, $4x - 4 = 24$.

C $w - x + y; w = 6, x = 5, y = 3$

$(6) - (5) + (3)$	Substitute 6 for w, 5 for x, and 3 for y.
$1 + 3$	Subtract.
4	Add.

When $w = 6$, $x = 5$, $y = 3$, $w - x + y = 4$.

D $x^2 - x; x = 9$

$(9)^2 - (9)$	Substitute 9 for each x.
$81 - 9$	Evaluate exponents.
72	Subtract.

When $x = 9$, $x^2 - x = 72$.

Math Talk
Mathematical Processes

Is $w - x + y$ equivalent to $w - y + x$? Explain any difference in the order the math operations are performed.

YOUR TURN

Evaluate each expression for $n = 5$.

4. $3(n + 1)$ _____ **5.** $4(n - 4) + 14$ _____ **6.** $6n + n^2$ _____

Evaluate each expression for $a = 3$, $b = 4$, and $c = -6$.

7. $ab - c$ _____ **8.** $bc + 5a$ _____ **9.** $a^2 - (b + c)$ _____

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Evaluating Real-World Expressions

You can evaluate expressions to solve real-world problems.

Math On the Spot
my.hrw.com

EXAMPLE 3

Real World

TEKS 6.7.A

The expression $1.8c + 32$ gives the temperature in degrees Fahrenheit for a given temperature in degrees Celsius c. Find the temperature in degrees Fahrenheit that is equivalent to 30 °C.

STEP 1 Find the value of c.

$c = 30$ °C

STEP 2 Substitute the value into the expression.

$1.8c + 32$

$1.8(30) + 32$ Substitute 30 for c.

$54 + 32$ Multiply.

86 Add.

86 °F is equivalent to 30 °C.

YOUR TURN

10. The expression $6x^2$ gives the surface area of a cube, and the expression x^3 gives the volume of a cube, where x is the length of one side of the cube. Find the surface area and the volume of a cube with a side length of 2 m.

$S =$ _____ m^2 ; $V =$ _____ m^3

11. The expression $60m$ gives the number of seconds in m minutes. How many seconds are there in 7 minutes?

_____ seconds

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Guided Practice

Evaluate each expression for the given value(s) of the variable(s).
(Examples 1 and 2)

1. $x - 7; x = 23$ _____

2. $3a - b; a = 4, b = 6$ _____

3. $\frac{8}{t}; t = 4$ _____

4. $9 + m; m = 1.5$ _____

5. $\frac{1}{2}w + 2; w = \frac{1}{9}$ _____

6. $5(6.2 + z); z = 3.8$ _____

7. The table shows the prices for games in Bella's soccer league. Her parents and grandmother attended a soccer game. How much did they spend if they all went together in one car? (Example 3)

Women's Soccer Game Prices	
Student tickets	$6
Nonstudent tickets	$12
Parking	$5

a. Write an expression that represents the cost of one carful of nonstudent soccer fans. Use x as the number of people who rode in the car and attended the game.

_____ is an expression that represents the cost of one carful of nonstudent soccer fans.

b. Since there are three attendees, evaluate the expression $12x + 5$ for $x = 3$.

12(____) + 5 = _____ + 5 = _____

The family spent _____ to attend the game.

8. Stan wants to add trim all around the edge of a rectangular tablecloth that measures 5 feet long by 7 feet wide. The perimeter of the rectangular tablecloth is twice the length added to twice the width. How much trim does Stan need to buy? (Example 3)

a. Write an expression that represents the perimeter of the rectangular tablecloth. Let l represent the length of the tablecloth and w represent its width. The expression would be _____.

b. Evaluate the expression $P = 2w + 2l$ for $l = 5$ and $w = 7$.

2(_____) + 2(_____) = 14 + _____ = _____

Stan bought _____ of trim to sew onto the tablecloth.

9. **Essential Question Follow Up** How do you know the correct order in which to evaluate algebraic expressions?

_____

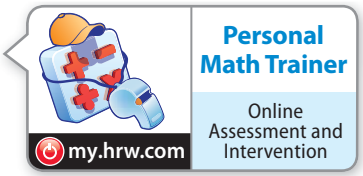
_____

_____

Name _____ Class _____ Date _____

11.2 Independent Practice

TEKS 6.7.A



10. The table shows ticket prices at the Movie 16 theater. Let a represent the number of adult tickets, c the number of children's tickets, and s the number of senior citizen tickets.

Movie 16 Ticket Prices	
Adults	$8.75
Children	$6.50
Seniors	$6.50

a. Write an expression for the total cost of tickets.

_____

_____

b. The Andrews family bought 2 adult tickets, 3 children's tickets, and 1 senior ticket. Evaluate your expression in part a to find the total cost of the tickets.

_____

_____

c. The Spencer family bought 4 adult tickets and 2 children's tickets. Did they spend the same as the Andrews family? Explain.

_____

11. The area of a triangular sail is given by the expression $\frac{1}{2}bh$, where b is the length of the base and h is the height. What is the area of a triangular sail in a model sailboat when $b = 12$ inches and $h = 7$ inches?

$A =$ _____ in.^2

12. Ramon wants to balance his checking account. He has $2,340 in the account. He writes a check for $140. He deposits a check for $268. How much does Ramon have left in his checking account? _____

13. **Look for a Pattern** Evaluate the expression $6x - x^2$ for $x = 0, 1, 2, 3, 4, 5,$ and 6. Use your results to fill in the table and describe any pattern that you see.

x	0	1	2	3	4	5	6
$6x - x^2$							

_____

_____

_____

_____

_____

_____

14. The kinetic energy (in joules) of a moving object can be calculated from the expression $\frac{1}{2}mv^2$, where m is the mass of the object in kilograms and v is its speed in meters per second. Find the kinetic energy of a 0.145-kg baseball that is thrown at a speed of 40 meters per second.

$E =$ _____ joules

15. The area of a square is given by x^2, where x is the length of one side. Mary's original garden was in the shape of a square. She has decided to double the area of her garden. Write an expression that represents the area of Mary's new garden. Evaluate the expression if the side length of Mary's original garden was 8 feet.

_____

16. The volume of a pyramid with a square base is given by the expression $\frac{1}{3}s^2h$, where s is the length of a side of the base and h is the height. Find the volume of a pyramid with a square base of side length 24 feet and a height of 30 feet.

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

17. **Draw Conclusions** Consider the expressions $3x(x - 2) + 2$ and $2x^2 + 3x - 12$.

a. Evaluate each expression for $x = 2$ and for $x = 7$. Based on your results, do you know whether the two expressions are equivalent? Explain.

_____

_____

_____

_____

_____

b. Evaluate each expression for $x = 1$. Based on your results, do you know whether the two expressions are equivalent? Explain.

_____

_____

_____

_____

_____

18. **Critique Reasoning** Marjorie evaluated the expression $3x + 2$ for $x = 5$ as shown:

$$3x + 2 = 35 + 2 = 37$$

What was Marjorie's mistake? What is the correct value of $3x + 2$ for $x = 5$?

_____

_____

_____

LESSON

11.3 Generating Equivalent Expressions

TEKS **Expressions, equations, and relationships—6.7.D** Generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties. *Also 6.7.C*

ESSENTIAL QUESTION

How can you identify and write equivalent expressions?

EXPLORE ACTIVITY 1 Real World TEKS 6.7.C

Identifying Equivalent Expressions

One way to test whether two expressions might be equivalent is to evaluate them for the same value of the variable.

Match the expressions in List A with their equivalent expressions in List B.

List A	List B
$5x + 65$	$5x + 1$
$5(x + 1)$	$5x + 5$
$1 + 5x$	$5(13 + x)$

A Evaluate each of the expressions in the lists for $x = 3$.

List A	List B
$5(3) + 65 =$ ☐	$5(3) + 1 =$ ☐
$5(3 + 1) =$ ☐	$5(3) + 5 =$ ☐
$1 + 5(3) =$ ☐	$5(13 + 3) =$ ☐

B Which pair(s) of expressions have the same value for $x = 3$?

_____

_____

C How could you further test whether the expressions in each pair are equivalent?

_____

D Do you think the expressions in each pair are equivalent? Why or why not?

_____

_____

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. **Error Analysis** Lisa evaluated the expressions $2x$ and x^2 for $x = 2$ and found that both expressions were equal to 4. Lisa concluded that $2x$ and x^2 are equivalent expressions. How could you show Lisa that she is incorrect?

_____

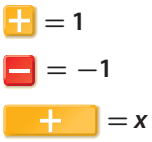
_____

EXPLORE ACTIVITY 2 Real World TEKS 6.7.C

Modeling Equivalent Expressions

You can also use models to determine if two expressions are equivalent. *Algebra tiles* are one way to model expressions.

Algebra Tiles

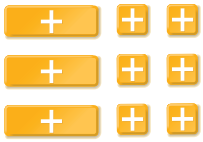


Determine if the expression $3(x + 2)$ is equivalent to $3x + 6$.

A Model each expression using algebra tiles.

$3(x + 2)$

$3x + 6$



B The model for $3(x + 2)$ has _____ x tiles and _____ 1 tiles.

The model for $3x + 6$ has _____ x tiles and _____ 1 tiles.

C Is the expression $3(x + 2)$ equivalent to $3x + 6$? Explain.

_____

_____

Reflect

2. Use algebra tiles to determine if $2(x - 3)$ is equivalent to $2x - 3$. Explain your answer.

_____

_____

Writing Equivalent Expressions Using Properties

Properties of operations can be used to identify equivalent expressions.

Properties of Operations	Examples
Commutative Property of Addition: When adding, changing the order of the numbers does not change the sum.	$3 + 4 = 4 + 3$
Commutative Property of Multiplication: When multiplying, changing the order of the numbers does not change the product.	$2 \times 4 = 4 \times 2$
Associative Property of Addition: When adding more than two numbers, the grouping of the numbers does not change the sum.	$(3 + 4) + 5 = 3 + (4 + 5)$
Associative Property of Multiplication: When multiplying more than two numbers, the grouping of the numbers does not change the product.	$(2 \times 4) \times 3 = 2 \times (4 \times 3)$
Distributive Property: Multiplying a number by a sum or difference is the same as multiplying by each number in the sum or difference and then adding or subtracting.	$6(2 + 4) = 6(2) + 6(4)$ $8(5 - 3) = 8(5) - 8(3)$
Identity Property of Addition: Adding zero to a number does not change its value.	$9 + 0 = 9$
Identity Property of Multiplication: Multiplying a number by one does not change its value.	$1 \times 7 = 7$
Inverse Property of Addition: The sum of a number and its opposite, or additive inverse, is zero.	$-3 + 3 = 0$

Math On the Spot
my.hrw.com

Math Talk
Mathematical Processes
What property can you use to write an expression that is equivalent to $0 + c$? What is the equivalent expression?

EXAMPLE 1

TEKS 6.7.D

Use a property to write an expression that is equivalent to $x + 3$.

The operation in the expression is addition.

You can use the Commutative Property of Addition to write an equivalent expression: $x + 3 = 3 + x$

YOUR TURN

For each expression, use a property to write an equivalent expression. Tell which property you used.

3. $(ab)c =$ _____

4. $3y + 4y =$ _____

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Identifying Equivalent Expressions Using Properties

Math On the Spot
my.hrw.com

Animated Math
my.hrw.com

EXAMPLE 2

Real World

TEKS 6.7.C

Use the properties of operations to determine if the expressions are equivalent.

A $3(x - 2); 3x - 6$

$3(x - 2) = 3x - 6$ Distributive Property

$3(x - 2)$ and $3x - 6$ are equivalent expressions.

B $2 + x; \frac{1}{2}(4 + x)$

$\frac{1}{2}(x + 4) = \frac{1}{2}x + 2$ Distributive Property

$= 2 + \frac{1}{2}x$ Commutative Property

$2 + x$ does not equal $2 + \frac{1}{2}x$.

They are not equivalent expressions.

Math Talk
Mathematical Processes

Explain how you could use algebra tiles to represent the Distributive Property in A.

YOUR TURN

Use the properties of operations to determine if the expressions are equivalent.

5. $6x - 8; 2(3x - 5)$

_____

_____

6. $2 - 2 + 5x; 5x$

_____

_____

7. Jamal bought 2 packs of stickers and 8 individual stickers. Use x to represent the number of stickers in a pack of stickers and write an expression to represent the number of stickers Jamal bought. Is the expression equivalent to $2(4 + x)$? Check your answer with algebra tile models.

_____

_____

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Generating Equivalent Expressions

Parts of an algebraic expression		
terms	The parts of the expression that are separated by + or − signs	$12 + 3y^2 + 4x + 2y^2 + 4$
coefficients	Numbers that are multiplied by at least one variable	$12 + 3y^2 + 4x + 2y^2 + 4$
like terms	Terms with the same variable(s) raised to the same power(s)	$12 + 3y^2 + 4x + 2y^2 + 4$

Math On the Spot
my.hrw.com

When an expression contains like terms, you can use properties to combine the like terms into a single term. This results in an expression that is equivalent to the original expression.

EXAMPLE 3

Real World

TEKS 6.7.D

Combine like terms.

A $6x^2 - 4x^2$

$6x^2$ and $4x^2$ are like terms.

$6x^2 - 4x^2 = x^2(6 - 4)$ Distributive Property

$= x^2(2)$ Subtract inside the parentheses.

$= 2x^2$ Commutative Property of Multiplication

$6x^2 - 4x^2 = 2x^2$

Math Talk
Mathematical Processes
Write 2 terms that can be combined with $7y^4$.

B $3a + 2(b + 5a)$

$3a + 2(b + 5a) = 3a + 2b + 2(5a)$ Distributive Property

$= 3a + 2b + (2 \cdot 5)a$ Associative Property of Multiplication

$= 3a + 2b + 10a$ Multiply 2 and 5.

$= 3a + 10a + 2b$ Commutative Property of Addition

$= (3 + 10)a + 2b$ Distributive Property

$= 13a + 2b$ Add inside the parentheses.

$3a + 2(b + 5a) = 13a + 2b$

☒ $y + 11x - 7x + 7y$

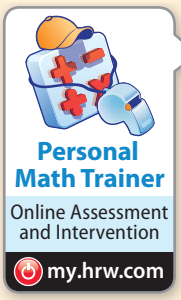
y and 7y are like terms; 11x and 7x are like terms.

$y + 11x - 7x + 7y = y + 7y + 11x - 7x$ Commutative Property

$= y(1 + 7) + x(11 - 7)$ Distributive Property

$= 8y + 4x$ Commutative Property

$y + 11x - 7x + 7y = 8y + 4x$



YOUR TURN

Combine like terms.

8. $8y - 3y =$ _____

9. $6x^2 + 4(x^2 - 1) =$ _____

10. $4a^5 - 2a^5 + 4b + b =$

_____

11. $8m + 14 - 12 + 4n =$

_____

Guided Practice

1. Evaluate each of the expressions in the list for $y = 5$. Then, draw lines to match the expressions in List A with their equivalent expressions in List B. (Explore Activity 1)

List A	List B
$4 + 4y =$ _____	$4y - 4 =$ _____
$4(y - 1) =$ _____	$4(y + 1) =$ _____
$4y + 1 =$ _____	$1 + 4y =$ _____

2. Determine if the expressions are equivalent by comparing the models. (Explore Activity 2) _____

$x - 4$ $\quad$ $2(x - 2)$

For each expression, use a property to write an equivalent expression. Tell which property you used. (Example 1)

3. $ab =$ _____

_____

4. $5(3x - 2) =$ _____

_____

Use the properties of operations to determine if each pair of expressions is equivalent. (Example 2)

5. $\frac{1}{2}(4 - 2x); 2 - 2x$ _____

6. $\frac{1}{2}(6x - 2); 3 - x$ _____

Combine like terms. (Example 3)

7. $32y + 12y =$ _____

8. $12 + 3x - x - 12 =$ _____

ESSENTIAL QUESTION CHECK-IN

9. Describe two ways to write equivalent algebraic expressions.

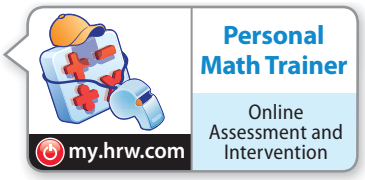
_____

_____

Name _____ Class _____ Date _____

11.3 Independent Practice

TEKS 6.7.D, 6.7.C



For each expression, use a property to write an equivalent expression. Tell which property you used.

10. $cd =$ _____

_____

11. $x + 13 =$ _____

_____

12. $4(2x - 3) =$ _____

_____

13. $2 + (a + b) =$ _____

_____

14. Draw algebra tile models to prove that $4 + 8x$ and $4(2x + 1)$ are equivalent.

_____ _____

Combine like terms.

15. $7x^4 - 5x^4 =$ _____

16. $32y + 5y =$ _____

17. $6b + 7b - 10 =$ _____

18. $2x + 3x + 4 =$ _____

19. $y + 4 + 3(y + 2) =$ _____

20. $7a^2 - a^2 + 16 =$ _____

21. $3y^2 + 3(4y^2 - 2) =$ _____

22. $z^2 + z + 4z^3 + 4z^2 =$ _____

23. $0.5(x^4 - 3) + 12 =$ _____

24. $\frac{1}{4}(16 + 4p) =$ _____

25. **Justify Reasoning** Is $3x + 12 - 2x$ equivalent to $x + 12$? Use two properties of operations to justify your answer.

_____

_____

_____

26. William earns $13 an hour working at a movie theater. Last week he worked h hours at the concession stand and three times as many hours at the ticket counter. Write and simplify an expression for the amount of money William earned last week.

_____

27. **Multiple Representations** Use the information in the table to write and simplify an expression to find the total weight of the medals won by the top medal-winning nations in the 2012 London Olympic Games. The three types of medals have different weights.

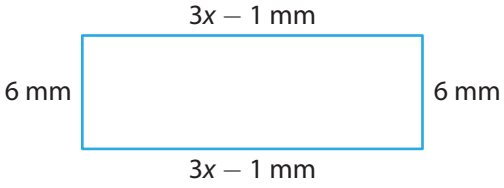
2012 Summer Olympics			
	Gold	**Silver**	**Bronze**
United States	46	29	29
China	38	27	23
Great Britain	29	17	19

_____

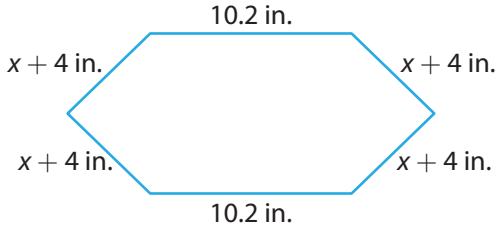
_____

Write an expression for the perimeters of each given figure. Simplify the expressions.

28. _____



29. _____



H.O.T. FOCUS ON HIGHER ORDER THINKING

30. **Problem Solving** Examine the algebra tile model.

a. Write two equivalent expressions for the model. _____

b. **What If?** Suppose a third row of tiles identical to the ones above is added to the model. How does that change the two expressions?

_____

31. **Communicate Mathematical Ideas** Write an example of an expression that cannot be simplified, and explain how you know that it cannot be simplified.

_____

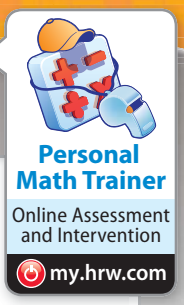
32. **Problem Solving** Write an expression that is equivalent to $8(2y + 4)$ that can be simplified.

_____

Work Area

MODULE QUIZ

Ready to Go On?



11.1 Modeling Equivalent Expressions

Write each phrase as an algebraic expression.

1. p divided by 6 _____

2. 65 less than j _____

3. the sum of 185 and h _____

4. the product of 16 and g _____

5. Let x represent the number of television show episodes that are taped in a season. Write an expression for the number of episodes taped in 4 seasons. _____

11.2 Evaluating Expressions

Evaluate each expression for the given value of the variable.

6. $8p; p = 9$ _____

7. $11 + r; r = 7$ _____

8. $4(d + 7); d = -2$ _____

9. $\frac{-60}{m}; m = 5$ _____

10. To find the area of a triangle, you can use the expression $b \times h \div 2$, where b is the base of the triangle and h is its height. What is the area of a triangle with a base of 6 and a height of 8? _____

11.3 Generating Equivalent Expressions

11. Draw lines to match the expressions in List A with their equivalent expressions in List B.

List A	List B
$7x + 14$	$7(1 + x)$
$7 + 7x$	$7x - 7$
$7(x - 1)$	$7(x + 2)$

ESSENTIAL QUESTION

12. How can you determine if two algebraic expressions are equivalent?

_____

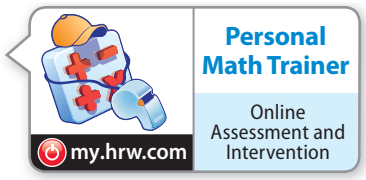
_____

_____

_____

MODULE 11 MIXED REVIEW

Texas Test Prep



Selected Response

1. Which expression represents the product of 83 and x?

Ⓐ $83 + x$

Ⓑ $83 \div x$

Ⓒ $83x$

Ⓓ $83 - x$

2. Which phrase describes the algebraic expression $\frac{r}{9}$?

Ⓐ the product of r and 9

Ⓑ the quotient of r and 9

Ⓒ 9 less than r

Ⓓ r more than 9

3. Rhonda was organizing photos in a photo album. She took 60 photos and divided them evenly among p pages. Which algebraic expression represents the number of photos on each page?

Ⓐ $p - 60$

Ⓑ $60 - p$

Ⓒ $\frac{p}{60}$

Ⓓ $\frac{60}{p}$

4. Using the algebraic expression $4n + 6$, what is the greatest whole-number value of n that will give you a result less than 100?

Ⓐ 22

Ⓑ 23

Ⓒ 24

Ⓓ 25

5. Evaluate $7w - 14$ for $w = 9$.

Ⓐ 2

Ⓑ 18

Ⓒ 49

Ⓓ 77

6. Katie has read 32% of a book. If she has read 80 pages, how many more pages does Katie have left to read?

Ⓐ 40

Ⓑ 170

Ⓒ 200

Ⓓ 250

7. The expression $12(x + 4)$ represents the total cost of CDs Mei bought in April and May at \$12 each. Which property is applied to write the equivalent expression $12x + 48$?

Ⓐ Associative Property of Addition

Ⓑ Associative Property of Multiplication

Ⓒ Commutative Property of Multiplication

Ⓓ Distributive Property

Gridded Response

8. When traveling in Europe, Bailey converts the temperature given in degrees Celsius to a Fahrenheit temperature by using the expression $9x \div 5 + 32$, where x is the Celsius temperature. Find the temperature in degrees Fahrenheit when it is 15 °C.

				•		
⓪	⓪	⓪	⓪		⓪	⓪
①	①	①	①		①	①
②	②	②	②		②	②
③	③	③	③		③	③
④	④	④	④		④	④
⑤	⑤	⑤	⑤		⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨