LESSON

7.2

Functions and Graphs

In Lesson 7.1, you learned that you can write rules for some of the coding grids. You can also write rules, often in the form of equations, to transform numbers into other numbers. One simple example is "Add one to each number." You can represent this rule with a table, an equation, a graph, or even a diagram.

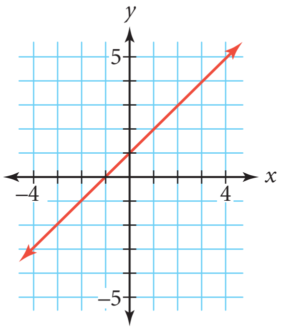
Table

Input x	Output y
7	8
-47	-46
10.28	11.28
x	$x+1$

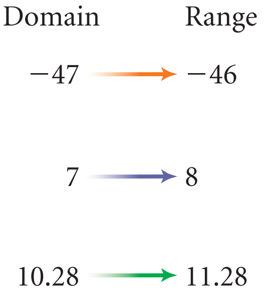
Equation

$y = x + 1$

Graph



Diagram



This rule turns 7 into 8, -47 into -46, 10.28 into 11.28, and x into $x + 1$.

When you explored relations in previous chapters, you used recursive routines, graphs, and equations to relate input and output data. To tell whether a relationship between input and output data is a function, there is a test that you can apply to the relation's graph on the *xy*-plane.

The Spanish painter Pablo Picasso (1881–1973) was one of the originators of the art movement Cubism. Cubists were interested in creating a new visual language, translating realism into a different way of seeing.

Investigation
Testing for Functions

In this investigation you will use various kinds of evidence to determine whether relations are functions.

Step 1 Each table represents a relation. Based on the tables, which relations are functions and which are not? Give reasons for your answers.

Table 1

Input x	Output y
-2	-3
-1	-1
0	1
1	3
2	5
3	7
4	9

Table 2

Input x	Output y
4	-2
1	-1
0	0
1	1
4	2
9	3
16	4

Table 3

Input x	Output y
-2	0.44
-1	0.67
0	1
1	1.5
2	2.25
3	3.37
4	5.06

Table 4

Input x	Output y
-2	-3
-1	-5
1	-1
1	-3
2	-10
3	-2
3	-8

Step 2 Each algebraic statement below represents a relation. Based on the equations, which relations are functions and which are not? Give reasons for your answers.

Statement 1 $y = 1 + 2x$

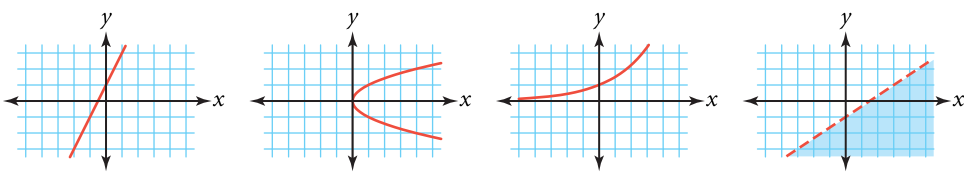
Statement 2 $y^2 = x$

Statement 3 $y = 1.5^x$

Statement 4 $y < -1 + \frac{2}{3}x$

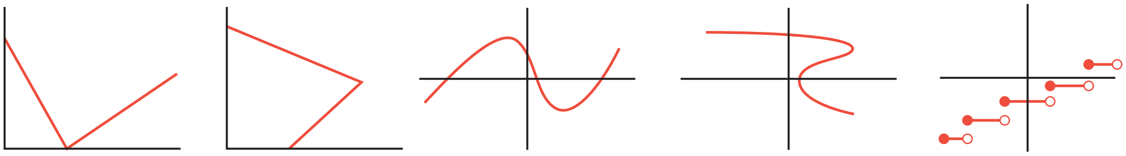
Step 3 Each graph below represents a relation. Move a vertical line, such as the edge of a ruler, from side to side on the graph. Based on the graph and your vertical line, which relations are functions and which are not? Give reasons for your answers.

Graph 1 **Graph 2** **Graph 3** **Graph 4**



Step 4 Use your results in Step 3 to write a rule explaining how you can determine whether a relation is a function, based only on its graph.

A function is a relation between input and output values. Each input has exactly one output. The **vertical line test** helps you determine if a relation is a function. If all possible vertical lines cross the graph once or not at all, then the graph represents a function. The graph does not represent a function if you can draw even one vertical line that crosses the graph two or more times.



Function   Not a function   Function   Not a function   Function

You have learned many forms of linear equations. In the example you will see whether all lines represent functions.

EXAMPLE

Name the form of each linear equation or inequality, and use a graph to explain why it is or is not a function.

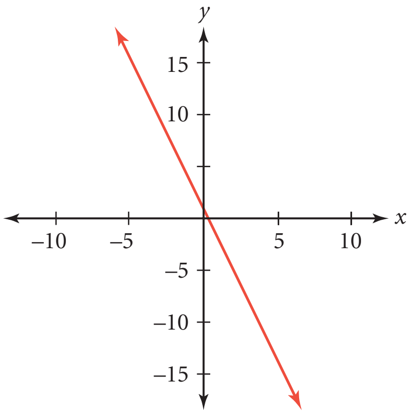
a. $y = 1 - 3x$ **b.** $y = 0.5x + 2$ **c.** $y = \frac{3}{4}x$ **d.** $2x + 3y = 6$

e. $y = 5 + 2(x - 8)$ **f.** $y = 7$ **g.** $x = 9$ **h.** $2x - 4y \leq 12$

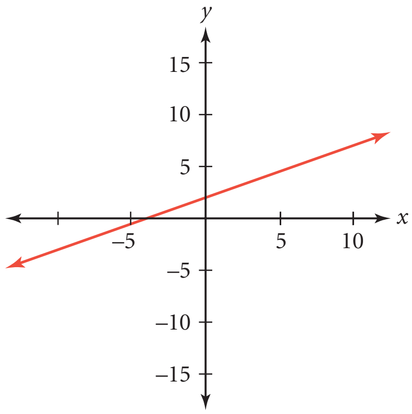
▶ Solution

Each equation is written in one of the forms you have learned in this course. If you graph the equations, you can see that all of them except the graphs for parts g and h pass the vertical line test. So all the equations represent functions except for the ones in parts g and h.

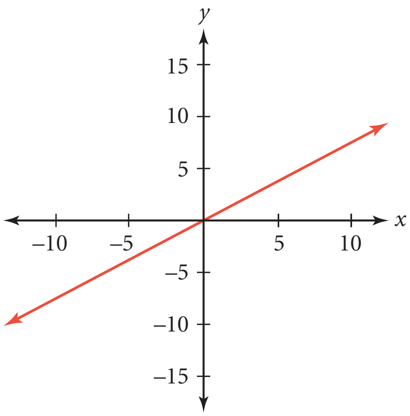
a. This equation is in the intercept form $y = a + bx$.



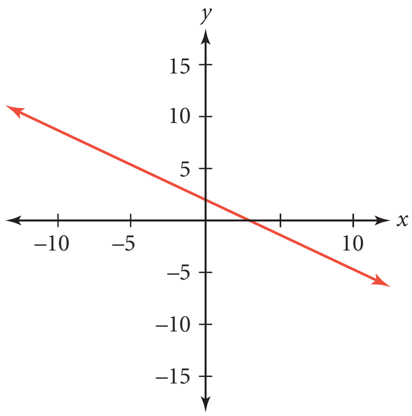
b. This equation is in the slope-intercept form $y = mx + b$.



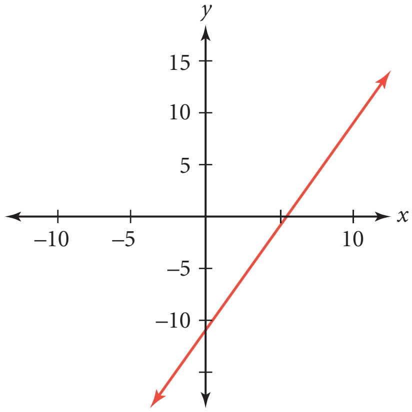
c. This equation is a direct variation in the form $y = kx$.



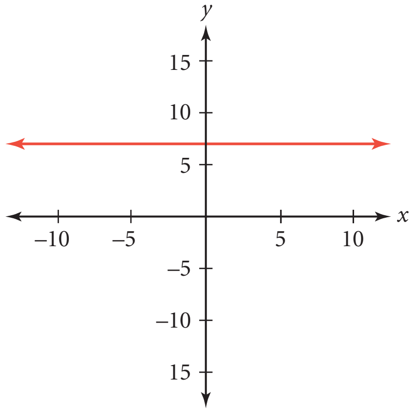
d. This equation is in the standard form $ax + by = c$.



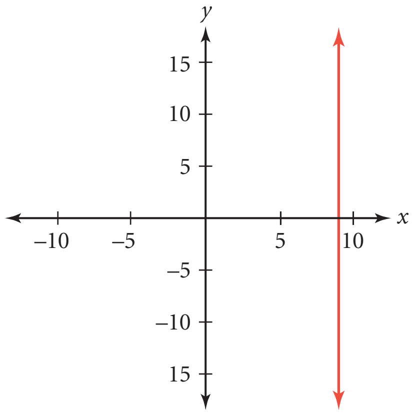
e. This equation is in the point-slope form $y = y_1 + b(x - x_1)$.



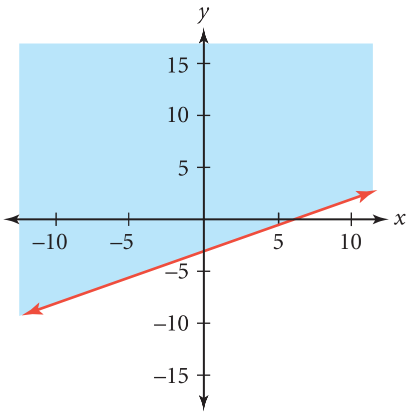
f. This equation is a horizontal line in the form $y = k$.



g. This equation is a vertical line in the form $x = k$.



h. The boundary of this inequality, $2x - 4y = 12$, is in the standard form $ax + by = c$.



The graphs of $x = 9$ and $2x - 4y \leq 12$ fail the vertical line test. In both cases you can match infinitely many output values of y to a single input value of x. So, $x = 9$ and $2x - 4y \leq 12$ do not represent functions. In fact, graphs of all vertical lines and linear inequalities fail the vertical line test, and are therefore not functions. All nonvertical lines are functions.

As you work more with functions, you will be able to tell if a relationship is a function without having to consider its graph on the xy-plane. If the graph is shown, use the vertical line test. Otherwise, see if there is more than one output value for any single input value.

Carpenters use a tool called a "level" to determine if support beams are truly vertical.

EXERCISES

You will need your graphing calculator for Exercise **15.**

Practice Your Skills

1. Use the equations to find the missing entries in each table.

a. $y = 4.2 + 0.8x$ @

Input x	Output y
-4	
-1	
1.5	
6.4	
9	

b. $y = 1.2 - 0.8x$

Domain x	Range y
-4	
-1	
2.4	
	-7.6
	-10

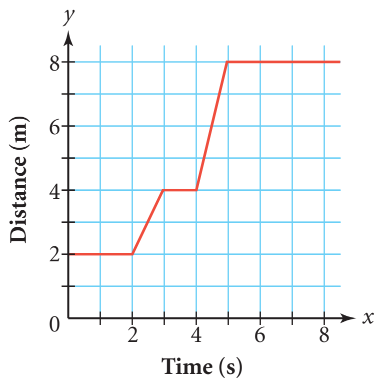
2. On the same set of axes, plot the points in the table and graph the equation in Exercise 1a.

3. On the same set of axes, plot the points in the table and graph the equation in Exercise 1b.

4. Use the tables and graphs in Exercises 1–3 to tell whether the relationships in Exercise 1 are functions. @

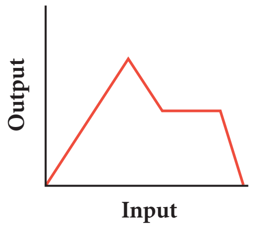
Reason and Apply

5. The graph at right describes another student's distance from you. What are the walking instructions for the graph? Does it represent a function? @

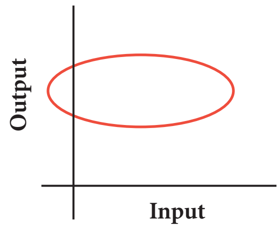


6. Find whether each graph below represents a function. Does it pass the vertical line test?

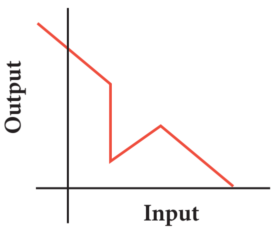
a.



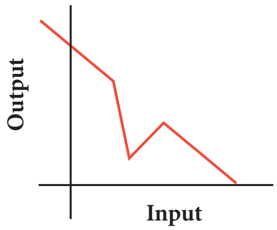
b.



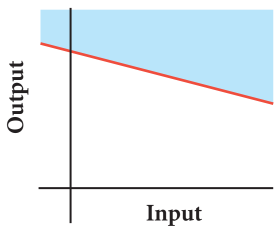
c. ⓗ



d.



e.



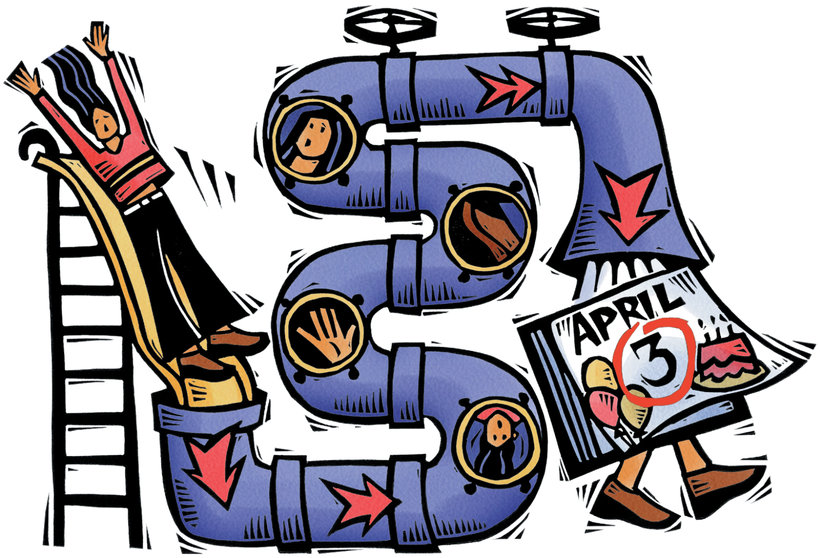
7. Does each relationship in the form (*input, output*) represent a function? If the relationship does not represent a function, find an example of one input that has two or more outputs. This is called a **counterexample.**

a. (*city, ZIP Code*)

b. (*person, birth date*)

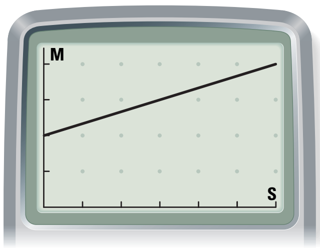
c. (*last name, first name*)

d. (*state, capital*)

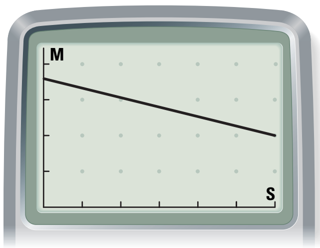


8. Here are the graphs of seven walks showing distance from a motion sensor.

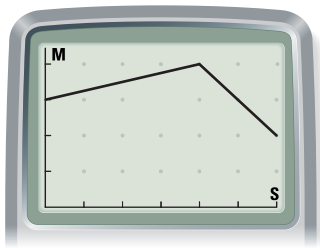
i.



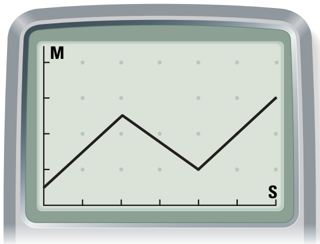
ii.



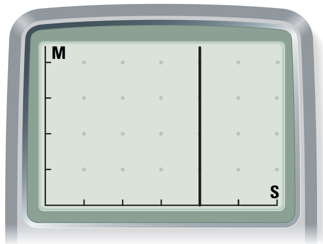
iii.



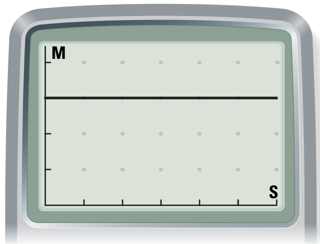
iv.



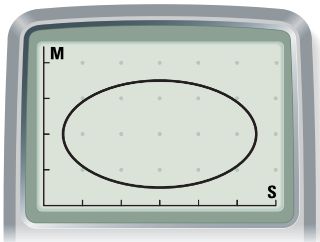
v.



vi.



vii.



a. Which graphs represent functions?

b. For which graphs is it not possible to write walking instructions?

c. What conclusion can you make?

9. Find whether each table of x- and y-values represents a function. Explain your reasoning.

a.

Domain x	Range y
0	5
1	7
3	10
7	9
5	7
4	5
3	8

b.

Domain x	Range y
3	7
4	9
8	4
5	5
9	3
11	9
7	6

c.

Domain x	Range y
2	8
3	11
5	12
7	3
9	5
8	7
4	11

10. On graph paper, draw a graph that is a function and has these three properties:

- Domain of x-values satisfying $-3 \leq x \leq 5$
- Range of y-values satisfying $-4 \leq y \leq 4$
- Includes the points $(-2, 3)$ and $(3, -2)$ @

11. On graph paper, draw a graph that is *not* a function and has these three properties:

- Domain of x-values satisfying $-3 \leq x \leq 5$
- Range of y-values satisfying $-4 \leq y \leq 4$
- Includes the points $(-2, 3)$ and $(3, -2)$

12. Complete the table of values for each equation. Let x represent domain values, and let y represent range values. Graph the points and find whether the equation describes a function. Explain your reasoning.

a. $x - 3y = 5$ @

x	2		-4		0	
y		1		-2		0

b. $y = 2x^2 + 1$

x	-2	3	0	-3	-1	
y						9

c. $x + y^2 = 2$

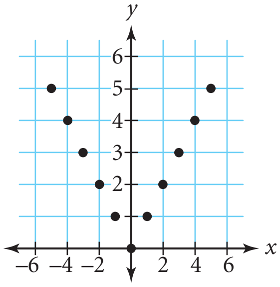
x	-7				-2	2
y		1	-2	-3		

d. $x + 2y = 4x$

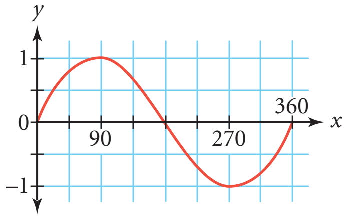
x						
y						

13. Identify all numbers in the domain and range of each graph.

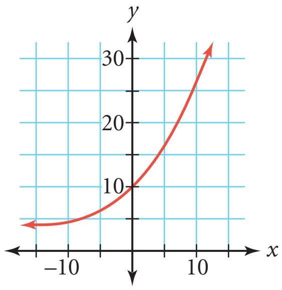
a.



b.



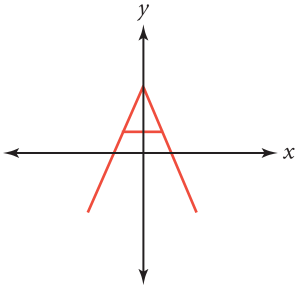
c.



14. Consider the capital letters in our alphabet.

a. Draw two capital letters that do not represent the graph of a function. Explain.

b. Draw two capital letters that do represent the graph of a function. Explain.



Review

15. If x represents actual temperature and y represents wind chill temperature, the equation

$$y = -29 + 1.4x$$

approximates the wind chill temperatures for a wind speed of 40 mi/h. Enter this equation into Y_1 on your calculator and find the requested x- and y-values.

a. What x-value gives a y-value of $-15°$? Explain how you use the calculator table function to find this answer.

b. Enter

$$y = -15$$

into Y_2 on your calculator. Graph both equations. Explain how to use the graph to answer 15a.

16. Show how you can use an undoing process to solve these equations.

a. $\dfrac{4(x-7)-8}{3} = 20$

b. $\dfrac{4.5}{x-3} = \dfrac{2}{3}$

17. Find the solution to each system.

a. $\begin{cases} y = 3x - 5 \\ y = -2.5x + 9 \end{cases}$

b. $\begin{cases} y = 2(x-4) + 15 \\ y = 15(x+5) - 12 \end{cases}$