Evaluating Limits Worksheet

(one method)

Evaluate the following limits without using a calculator.

1) $\lim_{x\to 3}\frac{2x^2-5x-3}{x-3}$

factor $2x^2-5x-3$

$2x^2+1x-6x-3$

$x(2x+1)-3(2x+1)$

$(2x+1)(x-3)$

$=\lim_{x\to 3}\frac{(2x+1)(x-3)}{(x-3)}$

$=\lim_{x\to 3} 2x+1 = 2(3)+1 = \boxed{7}$

2) $\lim_{x\to 2}\frac{x^4-16}{x-2}$

factor x^4-16

$(x^2+4)(x^2-4)$

$(x^2+4)(x+2)(x-2)$

$=\lim_{x\to 2}\frac{(x^2+4)(x+2)(x-2)}{(x-2)}$

$=\lim_{x\to 2}(x^2+4)(x+2) = (2^2+4)(2+2) = 8\cdot 4 = \boxed{32}$

3) $\lim_{x\to -1}\frac{x^4+3x^3-x^2+x+4}{x+1}$

polynomial long division

$$\begin{array}{r}
x^3+2x^2-3x+4 \\
x+1\,\big)\,x^4+3x^3-x^2+x+4 \\
-(x^4+x^3) \\
2x^2-x^2 \\
-(2x^2+2x) \\
-3x^2+x \\
-(-3x^2-3x) \\
4x+4 \\
-(4x+4) \\
0\ \checkmark
\end{array}$$

$=\lim_{x\to -1}\frac{(x+1)(x^3+2x^2-3x+4)}{x+1}$

$=\lim_{x\to -1} x^3+2x^2-3x+4 = -1+2+3+4 = \boxed{8}$

4) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$

$=\lim_{x\to 0}\frac{(\sqrt{x+4}-2)}{x}\frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)}$

$=\lim_{x\to 0}\frac{x+4-4}{x(\sqrt{x+4}+2)}$

$\lim_{x\to 0}\frac{x}{x(\sqrt{x+4}+2)}$

$=\lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}$

$=\frac{1}{\sqrt{4}+2}=\boxed{\frac{1}{4}}$

5) $\lim_{x\to 3}\frac{\sqrt{x+6}-x}{x-3}$

$=\lim_{x\to 3}\frac{(\sqrt{x+6}-x)}{(x-3)}\frac{(\sqrt{x+6}+x)}{(\sqrt{x+6}+x)} = \lim_{x\to 3}\frac{x+6-x^2}{(x-3)(\sqrt{x+6}+x)} = \lim_{x\to 3}\frac{-(x^2-x-6)}{(x-3)(\sqrt{x+6}+x)} = \lim_{x\to 3}\frac{-(x-3)(x+2)}{(x-3)(\sqrt{x+6}+x)}$

$=\lim_{x\to 3}\frac{-(x+2)}{\sqrt{x+6}+x} = \frac{-(3+2)}{\sqrt{3+6}+3} = \boxed{\frac{-5}{6}}$

6) $\lim_{x\to -2} \dfrac{\frac{1}{2}+\frac{1}{x}}{x+2}$ $= \lim_{x\to -2} \dfrac{\frac{x+2}{2x}}{\frac{x+2}{1}} = \lim_{x\to -2} \frac{x+2}{2x}\cdot\frac{1}{x+2} = \lim_{x\to -2} \frac{1}{2x} = \frac{1}{2(-2)} = \boxed{-\frac{1}{4}}$

7) $\lim_{x\to \frac{1}{2}} \dfrac{x^{-1}-2}{x-\frac{1}{2}}$ $= \lim_{x\to \frac{1}{2}} \dfrac{\frac{1}{x}-\frac{2}{1}}{\frac{x}{1}-\frac{1}{2}} = \lim_{x\to \frac{1}{2}} \dfrac{\frac{1-2x}{x}}{\frac{2x-1}{2}} = \lim_{x\to \frac{1}{2}} \frac{1-2x}{x}\cdot\frac{2}{2x-1} =$

$= \lim_{x\to \frac{1}{2}} \dfrac{-(2x-1)\cdot 2}{x(2x-1)} = \lim_{x\to \frac{1}{2}} \frac{-2}{x} = \frac{-2/1}{1/2} = \frac{-2}{1}\cdot\frac{2}{1} = \boxed{-4}$

8) $\lim_{x\to 3} \dfrac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$

$= \lim_{x\to 3} \dfrac{\frac{9-x^2}{9x^2}}{\frac{x-3}{1}} = \lim_{x\to 3} \frac{9-x^2}{9x^2}\cdot\frac{1}{x-3} = \lim_{x\to 3} \frac{-(x^2-9)}{9x^2}\,\frac{1}{x-3} = \lim_{x\to 3} \frac{-(x+3)(x-3)}{9x^2(x-3)}$

$= \lim_{x\to 3} \frac{-(x+3)}{9x^2} = \frac{-6}{9(9)} = \frac{-6}{81} = \boxed{\frac{-2}{27}}$

9) $\lim_{x\to 0} \dfrac{|x+2|-2}{|x|}$

$\lim_{x\to 0^+} \frac{|x+2|-2}{|x|}$ $= \lim_{x\to 0^+} \frac{x+2-2}{x} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$

$\lim_{x\to 0^-} \frac{|x+2|-2}{|x|}$ $= \lim_{x\to 0^-} \frac{x+2-2}{-x} = \lim_{x\to 0^-} \frac{x}{-x} = \lim_{x\to 0^-} -1 = -1$

because $\lim_{x\to 0^+} \neq \lim_{x\to 0^-}$

$\lim_{x\to 0} \frac{|x+2|-2}{x}$ is $\boxed{\text{undefined}}$

10) $\lim_{x\to 3} \dfrac{|x^2-9|}{|x-3|}$

$\lim_{x\to 3^+} \frac{|x^2-9|}{|x-3|} = \lim_{x\to 3^+} \frac{x^2-9}{x-3} = \lim_{x\to 3^+} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3^+} x+3 = 3+3 = 6$

$\lim_{x\to 3^-} \frac{|x^2-9|}{|x-3|} = \lim_{x\to 3^-} \frac{-(x^2-9)}{-(x-3)} = \lim_{x\to 3^-} \frac{-(x+3)(x-3)}{-(x-3)} = \lim_{x\to 3^-} x+3 = 3+3 = 6$

so $\lim_{x\to 3} \frac{|x^2-9|}{|x-3|} = \boxed{6}$

11) $\lim_{x\to 0}\left(\dfrac{5}{x^2+x}-\dfrac{5}{x}\right)$

$= \lim_{x\to 0} \frac{5}{x(x+1)} - \frac{5(x+1)}{x(x+1)} = \lim_{x\to 0} \frac{5-5(x+1)}{x(x+1)} = \lim_{x\to 0} \frac{5-5x-5}{x(x+1)} = \lim_{x\to 0} \frac{-5x}{x(x+1)} = \lim_{x\to 0} \frac{-5}{x+1} = \boxed{-5}$