

FRACTIONS



A **fraction** is defined as a ratio of two numbers, where the number at the bottom cannot be equal to zero.

$$\frac{a}{b} \text{ where } b \neq 0$$

In a fraction the number at the top is called the **numerator**, and the number at the bottom is called the **denominator**.

$$\frac{a}{b} = \frac{\text{numerator}}{\text{denominator}}$$

There are two different kinds of fractions, proper and improper. In a **proper** fraction the numerator (the number at the top) is less than the denominator (the number at the bottom). In an **improper** fraction, the numerator is greater than the denominator.

Proper fraction: numerator < denominator *Example:* $\frac{2}{3}$

Improper fraction: numerator > denominator *Example:* $\frac{5}{4}$

When you have an improper fraction, it is not always right or recommended to leave it as your final answer. It is always best to change an improper fraction to a mixed number. A **mixed number** is a whole number and a fraction together.

Mixed number: $C \frac{a}{b} \rightarrow C$ is a whole number and $\frac{a}{b}$ is a fraction

To change an improper fraction to a mixed number we need to divide the numerator by the denominator.

Let's take our previous example of an improper fraction to make it into a mixed number:

Example: $\frac{5}{4} \rightarrow$

$$\begin{array}{r}
 \text{Denominator} \swarrow \quad \searrow \text{Whole number} \\
 4 \overline{) 5} \\
 \underline{-4} \\
 1 \\
 \swarrow \text{Numerator}
 \end{array}
 \rightarrow 1\frac{1}{4}$$

When working with a fraction that contains large-value-numbers, it is always best to reduce the fraction to an equivalent fraction. An **equivalent fraction** is a fraction that has the same value but contains different numbers. To find an equivalent fraction, simply multiply the numerator and denominator by the same number.

Example: $\frac{3}{5} = \frac{3 \cdot 3}{5 \cdot 3} = \frac{9}{15}$ then $\frac{3}{5} = \frac{9}{15}$

In the case of reducing a fraction, or simplifying the fraction as you may also call it, we need to divide the numerator and denominator by the same number.

Example: $\frac{10}{25} = \frac{10 \div 5}{25 \div 5} = \frac{2}{5}$ therefore $\frac{10}{25} = \frac{2}{5}$

We have now found a fraction that is of the same value as the original one, but contains smaller digits and is easier to work with.

Now let's get into the basic operations of fractions.

1. Addition or subtraction of fractions with the same denominator.

When you need to add or subtract fractions that contain the same denominator, all you need to do is to add or subtract the numerators, depending on the problem, and keep the same denominator.

$$\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$$

Example: **Addition:** $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$

Subtraction: $\frac{5}{9} - \frac{3}{9} = \frac{5-3}{9} = \frac{2}{9}$

2. Addition or subtraction of fractions with different denominators.

To add or subtract fractions with different denominators, we must first find a common number between the two denominators making the problem easier to solve. This number is called **least common denominator (LCD)**. Using the LCD makes the same denominator for all the fractions and the operation will be easier to perform.

Suppose we have the following problem: $\frac{3}{5} + \frac{2}{9}$

To find the LCD, we must think of a common multiple for both denominators. Our denominators are 5 and 9 their multiples are as follow.

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, **45**, 50 ...

Multiples of 9: 9, 18, 27, 36, **45**, 54, 63, 72, 81, 90 ...

As you can see our LCD is 45 since it is the first multiple that both numbers have in common.

Now that we have found our LCD we can continue on with the operation. Rewrite both fractions to equivalent fractions with 45 as their denominator:

$$\frac{3}{5} + \frac{2}{9} \rightarrow \frac{27}{45} + \frac{10}{45}$$

We now have an addition of fractions with the same denominator and are now able to perform the operation:

$$\frac{27}{45} + \frac{10}{45} = \frac{37}{45}$$

3. Multiplication of fractions.

In multiplication of fractions, multiply numerators with numerators and denominators with denominators.

$$\frac{2}{5} \times \frac{4}{7} = \frac{2 \times 4}{5 \times 7} = \frac{8}{35}$$

4. Division of fractions.

To divide fractions we first need to invert the second fraction and then perform a multiplication of fractions.

$$\frac{1}{4} \div \frac{2}{3} \rightarrow \frac{1}{4} \times \frac{3}{2} = \frac{1 \times 3}{4 \times 2} = \frac{3}{8}$$

Invert the second fraction Multiply both fractions

5. Operations with whole numbers.

When dealing with whole numbers, we must first convert them into fractions. To do this simply put a 1 as the denominator and treat it as a fraction.

$$2 + \frac{3}{4} = \frac{2}{1} + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$$

Put a 1 as the denominator Obtained the LCD and perform the operation

6. Changing fractions to decimals.

To change fraction into decimals we simply need to divide the numerator by the denominator.

$$\frac{5}{4} \rightarrow 4 \overline{) 5} \begin{array}{r} 1.25 \\ -4 \\ \hline 10 \\ -8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array} \quad \text{Therefore: } \frac{5}{4} = 1.25$$

7. Fraction proportions.

Proportions are equivalent fractions that are missing a number, either the numerator or the denominator. We use cross multiplication to find the missing number.

$$\text{Example: } \frac{1}{4} = \frac{x}{3} \rightarrow (1)(3) = (x)(4) \rightarrow 3 = 4x \rightarrow x = \frac{3}{4}$$

8. Word problems containing fractions.

When working with word problems that involve fractions, follow the procedures described previously to solve the problem.

Example: A rectangular piece of material 3 feet wide by $12\frac{1}{2}$ feet long is cut into five equal strips. Find the length of each strip.

→ Find the length of each strip by cutting the piece length wise, so divide $12\frac{1}{2}$ by 5:

$$12\frac{1}{2} \div 5 = \frac{25}{2} \div \frac{5}{1} = \frac{25}{2} \times \frac{1}{5} = \frac{25}{10} = 2\frac{5}{10} = 2\frac{1}{2}$$

→ Therefore, each piece will be $2\frac{1}{2}$ feet.

FRACTIONS – EXERCISES

Reduce:

1. $-\frac{35}{70}$

2. $\frac{77}{121}$

3. $\frac{16}{56}$

4. $\frac{-9}{15}$

5. $\frac{15}{27}$

6. $\frac{40}{35}$

7. $\frac{18}{24}$

8. $\frac{48}{54}$

9. $\frac{19}{76}$

10. $\frac{20}{-75}$

Find x:

11. $\frac{2}{12} = \frac{9}{x}$

12. $\frac{3}{4} = \frac{x}{12}$

13. $\frac{15}{10} = \frac{x}{30}$

14. $\frac{1}{12} = \frac{3}{x}$

15. $\frac{55}{5} = \frac{22}{x}$

16. $\frac{7}{x} = \frac{14}{3}$

Perform the indicated operations and simplify your answer:

17. $\frac{8}{3} + \frac{9}{4}$

18. $\frac{11}{3} - \frac{5}{6}$

19. $\frac{1}{4} - \frac{2}{9}$

20. $\frac{1}{6} + \frac{3}{8} + \frac{1}{3}$

21. $\frac{13}{17} + \frac{2}{11}$

22. $\frac{5}{3} + 7$

23. $\frac{1}{3} + \frac{3}{7} + \frac{1}{6}$

24. $\frac{5}{3} - \frac{6}{7}$

25. $9 - \frac{3}{8}$

$$26. \frac{2}{3} \cdot \frac{1}{6}$$

$$27. 7 \div \frac{1}{9}$$

$$28. \frac{1}{8} \div 4$$

FRACTIONS - ANSWERS TO EXERCISES

$$1. \quad -\frac{35}{70} = -\frac{35 \div 35}{70 \div 35} = -\frac{1}{2}$$

$$2. \quad \frac{77}{121} = \frac{77 \div 11}{121 \div 11} = \frac{7}{11}$$

$$3. \quad \frac{16}{56} = \frac{16 \div 8}{56 \div 8} = \frac{2}{7}$$

$$4. \quad \frac{-9}{15} = -\frac{9}{15} = -\frac{9 \div 3}{15 \div 3} = -\frac{3}{5}$$

$$5. \quad \frac{15}{27} = \frac{15 \div 3}{27 \div 3} = \frac{5}{9}$$

$$6. \quad \frac{40}{35} = \frac{40 \div 5}{35 \div 5} = \frac{8}{7}$$

$$7. \quad \frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

$$8. \quad \frac{48}{54} = \frac{48 \div 6}{54 \div 6} = \frac{8}{9}$$

$$9. \quad \frac{19}{76} = \frac{19 \div 19}{76 \div 19} = \frac{1}{4}$$

$$10. \quad \frac{20}{-75} = -\frac{20}{75} = -\frac{20 \div 5}{75 \div 5} = -\frac{4}{15}$$

$$11. \quad \frac{2}{12} = \frac{9}{x}$$
$$2x = 108$$
$$x = 54$$

$$12. \quad \frac{3}{4} = \frac{x}{12}$$
$$4x = 36$$
$$x = 9$$

$$13. \quad \frac{15}{10} = \frac{x}{30}$$
$$10x = 450$$
$$x = 45$$

$$14. \quad \frac{1}{12} = \frac{3}{x}$$
$$x = 36$$

$$15. \quad \frac{55}{5} = \frac{22}{x}$$
$$55x = 110$$
$$x = 2$$

$$16. \quad \frac{7}{x} = \frac{14}{3}$$
$$14x = 21$$
$$x = \frac{3}{2}$$

$$17. \frac{8}{3} + \frac{9}{4} = \frac{32}{12} + \frac{27}{12} = \frac{59}{12}$$

$$18. \frac{11}{3} - \frac{5}{6} = \frac{22}{6} - \frac{5}{6} = \frac{17}{6}$$

$$19. \frac{1}{4} - \frac{2}{9} = \frac{9}{36} - \frac{8}{36} = \frac{1}{36}$$

$$20. \frac{1}{6} + \frac{3}{8} + \frac{1}{3} = \frac{4}{24} + \frac{9}{24} + \frac{8}{24} = \frac{21}{24} = \frac{7}{8}$$

$$21. \frac{13}{17} + \frac{2}{11} = \frac{143}{187} + \frac{34}{187} = \frac{177}{187}$$

$$22. \frac{5}{3} + 7 = \frac{5}{3} + \frac{21}{3} = \frac{26}{3}$$

$$23. \frac{1}{3} + \frac{3}{7} + \frac{1}{6} = \frac{14}{42} + \frac{18}{42} + \frac{7}{42} = \frac{39}{42}$$

$$24. \frac{5}{3} - \frac{6}{7} = \frac{35}{21} - \frac{18}{21} = \frac{17}{21}$$

$$25. 9 - \frac{3}{8} = \frac{72}{8} - \frac{3}{8} = \frac{69}{8} = 8\frac{5}{8}$$

$$26. \frac{2}{3} \cdot \frac{1}{6} = \frac{2}{18} = \frac{1}{9}$$

$$27. 7 \div \frac{1}{9} = \frac{7}{1} \div \frac{1}{9} = \frac{7}{1} \times \frac{9}{1} = 63$$

$$28. \frac{1}{8} \div 4 = \frac{1}{8} \div \frac{4}{1} = \frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$$

