Surface area and volume

This chapter deals with calculating the surface areas and volumes of right prisms and cylinders.

After completing this chapter you should be able to:

- solve problems involving the surface areas and volumes of right rectangular and triangular prisms
- calculate the surface areas and volumes of cylinders
- solve problems involving the surface areas and volumes of composite solids.

NSW Syllabus references: 5.2 M&G Area and surface area, 5.2 M&G Volume Outcomes: MA5.2-1WM, MA5.2-2WM, MA5.2-11MG, MA5.2-12MG MEASUREMENT & GEOMETRY - ACMMG242 (AC)

Diagnostic test

Questions 1 to 3 refer to the prism below.



The face that corresponds to the face *ABFE* is: 1

A	DCHG	В	BFGC
С	AEHD	D	DCGH

- **2** The face that corresponds to the face *CBFG* is:
 - A ABFE **B** AEHD C DAEH **D** DGCH
- **3** The face that corresponds to the face *ABCD* is: A EFHG **B** HEFG
 - D DCGH C EFGH
- **4** The surface area of the net shown is:



5 The surface area of the net shown is:



The surface area of this triangular prism is: 6



- **C** rectangular prism
 - **D** box



Exercise 2A

1 Calculate the area of each sector correct to 1 decimal place.







EXAMPLE 2

- a Draw a net for this rectangular prism, showing the lengths of its edges.
- **b** Calculate the surface area of the prism.





- **3** For each of the following rectangular prisms:
 - i Draw a net of each prism and mark its edge lengths.
- ii Calculate the surface area. b a c 4 cm 4.8 cm 3.6 m 5 cm 5.2 cm 7 cm 6.6 cm 1.8 m 2.4 m d f e 7 mm 10 mm 50 mm 10 cm 5 m 6 cm **EXAMPLE 3** a Draw a net of this triangular prism, marking its edge lengths. 5 cm 3.3 cm **b** Calculate the surface area of the prism. 8 cm 6 cm Solve/Think Apply Draw the net, identify the faces and transfer a the edge lengths from the solid to the net. Calculate the area of each face and sum these areas. 8 cm 4 cm 6 cm 5 cm 3.3 cm4 cm 5 cm SA = area of 2 triangles + area of 3 rectangles b $= (\frac{1}{2} \times 6 \times 3.3) \times 2 + 8 \times 4 + 8 \times 6 + 8 \times 5$ $= 139.8 \text{ cm}^2$
- 4 Calculate the surface area of each of the following triangular prisms.





5 For each triangular prism:

i Find the length of the unknown edge.



MEASUREMENT & GEOMETRY

Surface areas of right cylinders

The formula for the surface area of a cylinder can be developed by 'cutting' the Remember: Area of a circle is πr^2 . cylinder and laying it out flat. The net then gives a formula for the surface area. The curved part forms a rectangle of length $2\pi r$ and breadth *h*. A = 2(area of circle) + area of rectangle $= 2 \times \pi r^2 + 2\pi r \times h$ $2\pi r$ $=2\pi r^2+2\pi rh$ The surface area of a closed cylinder is: $A = 2\pi r^2 + 2\pi rh$ h h The surface area of a cylinder open at both $2\pi r$ ends is: $A = 2\pi rh$ Cut



Find the surface area of this closed cylinder.

B



Solve	Think	Apply
Surface area = $2\pi r^2 + 2\pi rh$ = $2\pi \times 6^2 + 2\pi \times 6 \times 15$ $\approx 791.7 \text{ cm}^2$ (1 decimal place)	Radius = 6 cm Height = 15 cm	For a cylinder closed at both ends: $SA = 2\pi r^2 + 2\pi rh$

Exercise 2B

1 Complete to find the surface area of this closed cylinder. Surface area = $2\pi r^2 + 2\pi$ ____

 $ea = 2\pi r^{2} + 2\pi \underline{\qquad} = 2\pi \times \underline{\qquad}^{2} + 2\pi \times 8.7 \times \underline{\qquad} = 2\pi \underbrace{\qquad}^{2} m^{2} (1 \text{ decimal place})$



2 Find the surface areas of these closed cylinders to the nearest whole number.



3 Find the surface areas of the following open cylinders.



- 4 a Determine how much paint is required to cover the outside of a cylindrical container 12 m long with diameter 10 m if each litre of paint covers 15 m².
 - **b** Which has the greater surface area: a cylinder of length 15 cm and radius 8 cm, or a cylinder of length 8 cm and radius 10 cm?

- **5** Find the surface area, correct to 1 decimal place where necessary, of:
 - a an open can with radius of 4 cm and height of 15 cm
 - **b** an open-ended pipe of 10 cm radius and 5 m long
 - c the closed solid shown below.



- 6 Determine the cost of painting the exterior walls and top of a cylindrical wheat silo that is 40 m high and 20 m in diameter, given that each litre of paint costs \$7.25 and covers 8 m².
- 7 Find the cost of making 125 cylindrical tennis ball containers that have diameter 7 cm and height 21 cm, given that the metal costs \$4.50 per square metre (metal base but open at the top).

EXAMPLE 3

An open cylinder of radius of 8 cm has a curved surface area of 1000 cm². Find its height.

Solve	Think	Apply
$1000 = 2 \times \pi \times 8 \times h$ = $16\pi \times h$ $h = \frac{1000}{16\pi}$ = 19.9 cm (1 decimal place)	To solve $1000 = 16\pi \times h$, divide both sides by 16π .	Substitute the given information into $SA = 2\pi rh$ and solve the resulting equation.

- 8 Find the height of an open cylinder of radius 10 cm and curved surface area of 2000 cm².
- **9** Find the radius of an open cylinder of height 5 cm and curved surface area of 1500 cm².



The volume of a right prism (or cylinder) is given by:

 $V = A \times h$

where A is the area of the base (or cross-sectional area) and h is the perpendicular height.



EXAMPLE 1



Exercise 2C



3 Calculate the area of the base and hence find the volume of each solid.





4 Complete to find the volume of this solid correct to 1 decimal place.

$$V = A \times h$$

$$= \frac{\Box}{360} \times \underline{\qquad} \times r^{2} \times h$$
$$= \frac{\Box}{360} \times \pi \times \underline{\qquad}^{2} \times \underline{\qquad}$$
$$= \underline{\qquad} \text{ cm}^{3}$$







EXAMPLE 1

Calculate the volume of this composite solid.



Solve	Think	Apply
<i>Cylinder:</i> $V = \pi r^2 \times h$	The solid is made up of a	Break the composite solid
$=\pi imes 5^2 imes 8$	cylinder and a cube.	into simpler solids and find
$= 628.318 \text{ cm}^3$	Cylinder:	the volume of each one
Cube: V = Ah	Radius = $10 \div 2 = 5$ cm	separately.
$= 10 \times 10 \times 10$	Height = 8 cm	Combine the volumes to
$= 1000 \text{ cm}^3$	Cube:	give the answer.
Total volume = $628.318 + 1000$	l = b = h = 10 cm	
$= 1628 \text{ cm}^3$ to nearest cm ³		

Exercise 2D

1 Complete to find the volume of this composite solid.



The solid is a _____ and a rectangular prism. *Cylinder:*

Diameter = 6 m Radius = _

$$V = \pi r^2 \times _$$

 $= \pi \times _^2 \times _$
 $= _ m^3$
Rectangular prism:
 $V = Ah$
 $= (12 \times _) \times 4$
 $= _ m^3$

Total volume = $_$ + $_$ = $_$ m³ to the nearest whole number

2 Calculate the volume of each composite solid.



Surface areas of composite solids

EXAMPLE 1



Exercise 2E





Problems with surface area and volume

Exercise 2F

- A sheet of cardboard 1200 mm by 1000 mm has squares of side-length 300 mm cut from each corner. The sides are folded up to form an open rectangular box.
 - **a** Calculate its internal surface area.
 - **b** What is the volume of the box?



- 2 A carport and workshop are covered by a flat rectangular roof 3.6 m by 11.2 m. All the rain that falls on the roof is collected in a water tank. If 3 mm of rain falls on the roof, how much water will be collected in the tank? $(1 \text{ m}^3 = 1000 \text{ L})$
- **3** The cross-section of this closed rainwater tank is shown beside it.



- a Calculate the area of this cross-section.
- **b** Hence calculate the volume of the tank.
- c What is the capacity of the tank if 1 m^3 holds 1000 L?
- **d** The tank is completely made from sheet steel that costs \$40/m². What is the cost of the steel to make this tank?
- **4** The diagram shows the design for a concrete driveway.
 - a Calculate its area.
 - **b** A concrete contractor charges \$70/m² to supply and lay concrete. How much will he charge for this job? Give the answer to the nearest dollar.
 - **c** If the concrete needs to be 100 mm deep, calculate the volume of concrete needed, in cubic metres.





- **5** The cylindrical roller for a cricket pitch is 1.5 m wide and has a radius of 0.3 m.
 - **a** Calculate the curved surface area of the roller.
 - **b** What is the minimum number of revolutions the roller would have to make to roll the cricket pitch once if the pitch is 20 m long and 3 m wide? (Ignore any revolutions needed to turn the roller around.)
- **6** A backyard swimming pool has dimensions as shown.
 - **a** Calculate the volume of the pool.
 - b How long will it take to fill the pool with water from a garden hose that can supply water at the rate of 7.5 L/min? (Use 1 m³ holds 1000 L of water.)



- c What is the cost of filling the pool if water costs \$2.75/kL?
- 7 A hollow iron pipe is 2 m long. Its external diameter is 10 cm and it is 1 cm thick. Calculate the weight of the pipe if iron weighs 8.2 g/cm³.
- 8 A fish tank has a rectangular base 40 cm by 20 cm. Water is poured in to a height of 24 cm.
 - **a** What is the volume of water in the tank?
 - **b** If a further 2 litres of water is poured into the tank, by how much will the water level rise?
- A pontoon with base 3 m by 3 m is floating on a lake. When a man swims out and climbs onto it the pontoon sinks 1 cm. If 1 L of water weighs 1 kg, what is the weight of the man? (*Hint:* Archimedes' principle tells us that the weight of the man is equal to the weight of water displaced.)
- **10** A 50 cm³ block of metal is made into wire of diameter 1 mm. How long will the wire be?



11 A greenhouse with the dimensions shown is to be covered on the top and sides only (not the front and back) with shade cloth. The shade cloth comes in 20 m rolls and is 1.8 m wide.



- a Calculate the number of linear metres of shade cloth needed.
- **b** How many rolls will be needed?

Language in mathematics

Johann Kepler (1571-1630)

Johann Kepler was born in the German town of Wurttemberg. As a child he was small and suffered from ill health, but he was recognised as being intelligent. He was given a scholarship to attend the University of Tubingen, where he studied first for the Lutheran ministry and then science. He studied under a master in astronomy who believed in, and taught, the Copernican theory that Earth rotated around its own axis and around the Sun. Kepler taught mathematics in Graz from 1594.



In 1600 he went to Prague and became assistant to Tycho Brahe, an important astronomer. After Brahe's death, Kepler succeeded him as astronomer and mathematician to the emperor. Kepler had access to Brahe's extensive records of observations and calculations.

Kepler believed in the Copernican theory, and became one of the founders of modern astronomy. He developed three fundamental laws of planetary motion, now known as Kepler's Laws, in 1609. These proposed, among other things, that the Sun was at the centre of our planetary system, and that the orbits of the planets were elliptical rather than circular. Sixty years later these laws helped Newton to develop his Universal Law of Gravitation.

Kepler also suggested that tides are caused by the Moon's gravitational pull on the seas. He produced tables giving the positions of the Sun, Moon and planets, which were used for about 100 years. In 1611 he proposed an improved refracting telescope, and later he suggested a reflecting telescope that was developed by Newton.

- **1** a How old was Kepler when he died?
 - **b** When and where did Kepler teach mathematics?
 - c Describe the development of Kepler's ideas concerning planetary motion.
 - **d** Research Kepler's three laws.
 - e For how long were Kepler's tables of positions of the Sun, Moon and planets used?
 - **f** How are tides formed?
- **2** Rearrange these words to form a sentence.
 - **a** a circle a semicircle A half is of.
 - **b** a is of quarter quadrant A circle a.
 - c may way than Composite more in areas one be found.
- **3** Use every third letter to find the sentence.

WD T R F H T G E H Y A U J R N H E G B A V F O E D F S WA A Z R D F H H J O L P MO E B Q A U Z D S F Y O I J R B WA Q A K C G I H J T I I E O P I L L S G F H D E A S K L A X F V B T H Q H S O E Y A P E F R H K O I P D N M U A E C S D T C G O H N F B E T W X H A U E I O D A G I B H A J K G N H O D S N W E A D F L T Y S

Terms



8 A lidded wooden box, 15 cm × 8.5 cm × 6 cm, is to be lacquered inside and out with two coats of lacquer. Ignoring the thickness of the wood, the total area to be lacquered is:

A 537 cm² **B** 2148 cm² **C** 1074 cm² **D** 2685 cm²

If you have any difficulty with these questions, refer to the examples and questions in the sections listed in the table.

Question	1–3	4	5	6	7	8
Section	А	В	С	D	Е	F

2A Review set



5 Calculate the surface area and volume of this solid.



2B Review set

 Deborah's family room is shown opposite. Calculate the cost of carpet-tiling the room if the carpet tiles costs \$119.80 per square metre.





⊢ 3 m

5 m

9 cm

3 m

4 m

2 m

5 cm

- 2 A door wedge shaped as shown is to be painted. What is the total area to be painted?
- **3** Calculate the surface area and volume of a closed cylinder with diameter 2.4 m and height 1.8 m.
- 4 Calculate the surface area of this solid.



6 The cross-section of this rainwater tank is shown beside it.





9 cm

- **a** Calculate the area of this cross-section.
- **b** Hence calculate the volume of the tank.
- **c** What is the capacity of the tank if 1 m³ holds 1000 L?
- **d** The tank was made from sheet steel that costs \$45/m². What was the cost, to the nearest dollar, of the steel used to make this tank?

2C Review set

1 Calculate the area of each shape correct to 1 decimal place.



5 A hollow steel pipe is 5 m long. Its external diameter is 20 cm and it is 1.5 cm thick. Calculate the weight of the pipe to the nearest gram given that steel weighs 8.2 g/cm³.



2D Review set



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