

Arithmetic and Geometric Sequences

A _____ is a list of numbers in a particular order. Ex: 3, 6, 9, 12, ...

First term- a_1

Second term- a_2

And so on....

Many sequences have patterns. The two types of sequences we will be studying are arithmetic and geometric.

Arithmetic Sequences

An arithmetic sequence is a sequence where each term is found by adding a constant to the previous term. This constant is called the **common difference (d)**.

1. Find the common difference of the following sequence: 3, 6, 9, 12, ...

2. Find the common difference of the following sequence: 55, 49, 43,...

Let's take a detour and look at two different types of formulas: **recursive** and **explicit**.

In a recursive formula, each term is found by doing something to the term immediately in front of that term.

For example, take the arithmetic sequence 2, 4, 6, 8,....
Each term is found by adding _____ to the term before it.

A recursive for this sequence would be $a_n = a_{n-1} + 2$

Let's check it:

$$a_2 =$$

$$a_3 =$$

Use this recursive formula to find a_6 .

Could we use this formula to find a_{100} ?

What are some drawbacks to recursive formulas?

Thankfully, there are explicit formulas. An explicit formula helps you to find any term in a sequence quickly, as long as you know the first term.

The explicit formula for an arithmetic sequence is $a_n = a_1 + d(n - 1)$

$a_1 = \text{first term}$

$n = \text{number of term you are looking for}$

$d = \text{common difference}$

Write an equation for the n th term of the arithmetic sequence 8, 17, 26, 35,...

3. Write an explicit equation for the n th term of the sequence 1.3, 3.8, 6.3,...

Now, we can use our knowledge of creating explicit formulas to find any term

4. Find the 50th term for the sequence 31, 24, 17,...

Given the sequence defined by the function $a_{n+1} = a_n + 12$ with $a_1 = 4$. Write an explicit function rule.

Geometric Sequences

-A geometric sequence is a sequence where each term is found by multiplying the previous term by a constant called the common ratio (r).

The common ratio can be found by dividing any term by its previous term.

5. Find the common ratio of the following sequence 8, 20, 50, 125, ...
6. Find the common ratio of 1.25, -1.5, 1.8,...
7. Find the common ration of 405, 135, 45, ...
8. Write a recursive formula for the following sequence: 3, 6, 12, 24, 48, 96
9. Write a recursive formula for the following sequence: 81, 108, 144. Then use this recursive formula to find the 5th term.

Like arithmetic sequences, we can write explicit formulas for geometric sequences.

The explicit formula for a geometric sequence is $a_n = a_1 \cdot r^{n-1}$

$a_1 = \text{first term}$

$n = \text{number of term you are looking for}$

$r = \text{common ratio}$

Write an equation for the n th term of the geometric sequence 3, 12, 48, 192,...

Write an equation for the n th term of the geometric sequence 4, 6, 9, 13.5,...

Find the eighth term of a geometric sequence for which $a_1 = -3$ and $r = -2$.

Given the sequence defined by the function $a_{n+1} = \frac{3}{4}a_n$ with $a_1 = 424$. Write an explicit function rule.

Practice/Homework:Solve on a separate sheet of paper

For each sequence, state if it is arithmetic, geometric, or neither.

- 1, 3, 6, 10, 15
- 40, 43, 46, 49, 52
- $4, \frac{13}{3}, \frac{14}{3}, 5, \frac{16}{3}$
- $-4, 12, -36, 108, -324$
- 4, 16, 36, 64, 100
- $-29, -34, -39, -44, -49$

Find the next four terms of each arithmetic sequence using a recursive formula.

- 6, -2, 2,...
- 31, 24, 17,...

Find the first five terms of each arithmetic sequence described.

- $a_1 = 2, d = 13$
- $a_1 = 6, d = -4$

Find the indicated term of each arithmetic sequence.

- $a_1 = 3, d = 7, n = 14$
- $a_1 = -4, d = -9, n = 20$
- $a_1 = 35, d = 3, n = 101$
- $a_1 = 20, d = 4, n = 81$
- $a_1 = 5, d = \frac{1}{3}, n = 12$
- $a_1 = \frac{5}{2}, d = -\frac{3}{2}, n = 11$

Find the next two terms of each geometric sequence using a recursive formula.

- 405, 135, 45,...
- 81, 108, 144,...

Find the first five terms of each geometric sequence described

- $a_1 = 2, r = -3$
- $a_1 = 243, r = \frac{1}{3}$

Find the indicated term of each geometric sequence.

- $a_1 = \frac{1}{3}, r = 3, n = 8$
- $a_1 = \frac{1}{64}, r = 4, n = 9$
- $a_1 = 16,807, r = \frac{3}{7}, n = 6$
- $a_1 = 4096, r = \frac{1}{4}, n = 8$